DISCRETE MATHEMATICS GRADE 6

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Lessons adapted from Connected Mathematics Project

Works Cited:

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Accentuate the Negative* (2nd ed., Connected Mathematics). Upper Saddle River, NJ: Pearson.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *How Likely Is It?* (2nd ed., Connected Mathematics). Upper Saddle River, NJ: Pearson.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Filling and Wrapping* (2nd ed., Connected Mathematics). Upper Saddle River, NJ: Pearson.

Executive Summary

Discrete Math Unit Overview

This unit on Discrete Mathematics will build skills on probability, fractions, area, perimeter, volume, surface area, and prime numbers using tables, graphs, models, and manipulatives. The students will be involved in real world problem solving as well as hands-on learning. For example, students will use organized lists and tree diagrams to find theoretical probabilities.

MN State Standards Addressed:

6.1.1.5.
Factor whole numbers; express a whole number as a product of prime factors with exponents.

6.1.1.6.
Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

6.3.1.1
Calculate the surface area and volume of prisms and use appropriate units, such as cm^2 and cm^3. Justify the formulas used. Justification may involve decomposition, nets, or other models.

6.3.1.2
Calculate the area of quadrilaterals. Quadrilaterals include squares, rectangles, rhombuses, parallelograms, trapezoids, and kites. When formulas are used, be able to explain why they are valid.

6.4.1.1
Determine the sample space (set of possible outcomes) for a given experiment and determine which members of the sample space are related to certain events. Sample space may be determined by the use of tree diagrams, tables, or pictorial representations.

6.4.1.2

Determine the probability of an event using the ratio between the size of the event and the size of the sample space; represent probabilities as percents, fractions, and decimals between 0 and 1 inclusive. Understand that probabilities measure likelihood.

SAMPLE MCA QUESTIONS STUDENTS WILL BE ABLE TO SUCCESSFULLY ANSWER.

16. Eli has a cube with sides numbered 1-6 and a spinner with 3 equal sections labeled A, B, and C. He rolls the cube and spins the spinner. How many outcomes are possible?

What is the prime factorization of 630?

A. 2 x3 x 5 x 7
B. 2 x 3^2 x 5 x 7
C. 2 x 3^2 x 35
D. 2 x 5 x 7 x 9

The surface area of a cube is 384 square inches. What is the volume of the cube?

A. 8 cubic inches
B. 16 cubic inches
C. 256 cubic inches
D. 512 cubic inches

What is the greatest common factor of 48 and 64?

A. 2
B. 8
C. 16
D. 24

Table of Contents

Name: ______________________

Discrete Math Unit Pre-Test

1. What is the difference between area and perimeter?

Use figure B to answer the questions 2 and 3.

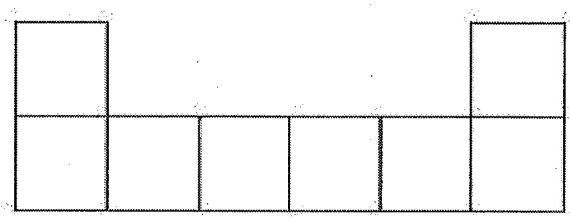
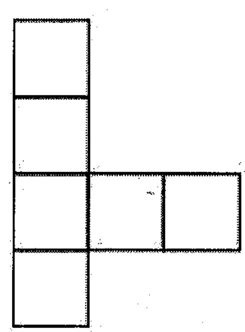


Figure B

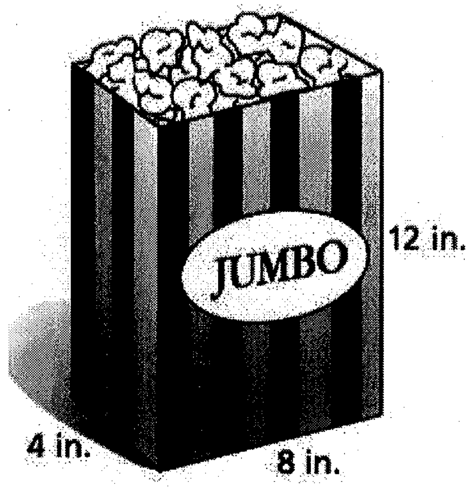
2. What is the area of figure B? Label your answer. ______________________

3. What is the perimeter of Figure B? Label your answer. ________________

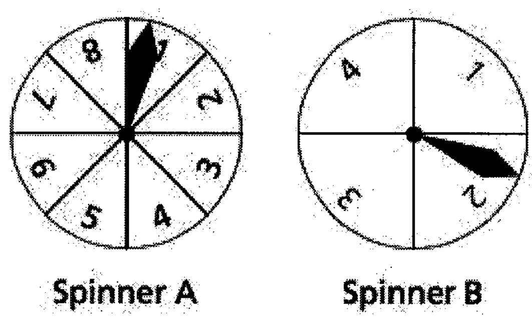
4. Could the net below be folded along the lines to form a box? If yes, explain how. If no, explain why not.



5. The Apple Theater concession sells two sizes of popcorn--a micro box and a jumbo box. About how many square inches of cardboard are needed to make the jumbo box? (there is no top on the box). You may use cubes if you would like.



Use the spinners below to answer question 5.



6. If Spinner A is spun first, then spinner B is spun, what are all of the possible number pairs that could be spun?

Lesson 1: Making Tree Diagrams to Find Probability

Mathematical Goals (Objective):

- Use organized lists and tree diagrams to find theoretical probabilities.

Launch:

Pose the question "I toss four coins, what are all of the possible outcomes?" Toss four coins two times and list the two outcomes. Give students time to work in a think-pair-share format. Inevitably some students will have more or less outcomes than others. Listen to strategies of students, if someone has a tree diagram have them explain it for the class.

After students have completed the discussion for the first coin problem. If nobody has a tree diagram ask how you could decide what options were on the first coin, then if it was Heads, what are my options for the second coin? Continue until a tree-diagram takes shape.

Explore:

Introduce the next problems for the students to do. Start with the question about the color combinations. "I wonder if I had 4 colors of paper and 3 colors of markers how many color combinations I could make?" "Any ideas on how I could do this? (tree diagram!!!!!)" After students share their color combination work you can check for understanding and let them work on the rest of the tree diagram problems. Help groups that might be stuck--or better yet guide students to explain how to they did the problem to the groups that might be stuck.

Share:

Students will share strategies and initial thoughts on the first problem about the coins. Teacher will guide them to a tree diagram if it doesn't naturally take place. Guide students to see that a tree diagram is like a map pointing out all of the outcomes. Students will also share their strategies and tree diagrams for the colored paper/colored markers problem. At the end of class the teacher will also give the students a closing question: How are tree-diagrams helpful in determining outcomes? Each table must come up with a response to share with the class before they may leave the classroom.

Summarize:

After walking around the room as students were working the teacher will summarize how students created tree diagrams. The teacher will then summarize the table's ideas before heading out the door. Today's lesson was about tree diagrams, they are a powerful strategy for solving mathematical problems like this.

7. Melissa is designing a birthday card for her sister. She has a blue, a yellow, a pink, and a green sheet of paper. She also has a black, a red, and a purple marker. Suppose Melissa chooses one sheet of paper and one marker at random.

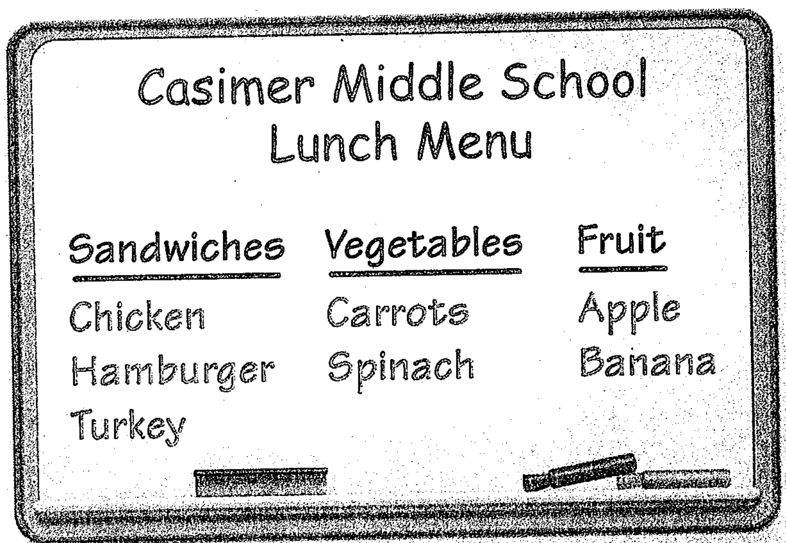
   a. Make a tree diagram to find all the possible color combinations.

   b. What is the probability that Melissa chooses pink paper and a red marker?

   c. What is the probability that Melissa chooses blue paper? What is the probability she does *not* choose blue paper?

   d. What is the probability that she chooses a purple marker?

8. Lunch at Casimer Middle School consists of a sandwich, a vegetable, and a fruit. Today there is an equal number of each type of sandwich, vegetable, and fruit. The students don't know what lunch they will get. Sol's favorite lunch is a chicken sandwich, carrots, and a banana.



   a. Make a tree diagram to determine how many different lunches are possible. List all the possible outcomes.

   b. What is the probability that Sol gets his favorite lunch? Explain your reasoning.

   c. What is the probability that Sol gets at least one of his favorite lunch items? Explain.

7

12. Silvia and Juanita are designing a game. In the game, you toss two number cubes and consider whether the sum of the two numbers is odd or even. They make a tree diagram of possible outcomes.

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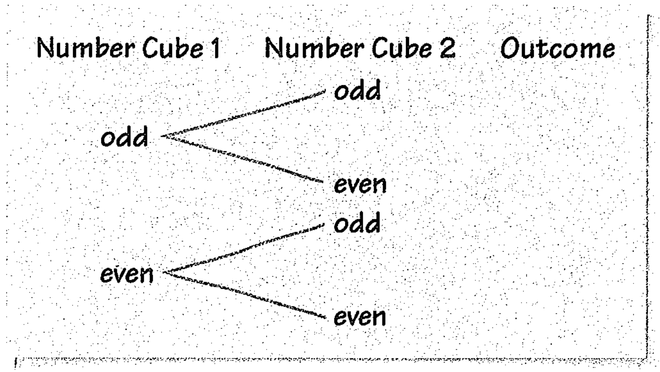


a. List all the outcomes.

b. Design rules for a two-player game that is fair.

c. Design rules for a two-player game that is not fair.

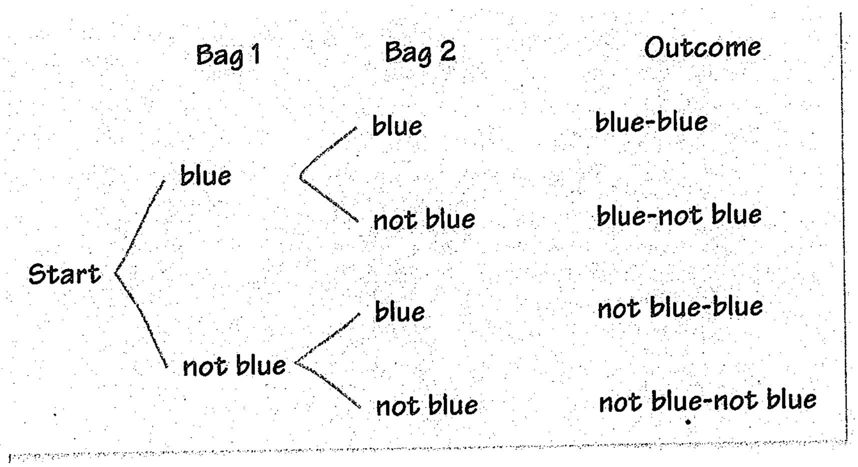
d. How is this situation similar to tossing two coins and seeing if the coins match or don't match?

36. Suppose you compete for the bonus prize on the *Gee Whiz Everyone Wins!* game in Problem 2.3. You choose one block from each of two bags. Each bag contains one red, one yellow, and one blue block.

a. Make a tree diagram to show all the possible outcomes.

b. What is the probability that you choose two blocks that are *not* blue?

c. Jason made the tree diagram shown below to find the probability of choosing two blocks that are *not* blue. Using his tree, what probability do you think Jason got?



d. Does your answer in part (b) match Jason's? If not, why do you think Jason gets a different answer?

Connections Extensions

2.3 Winning the Bonus Prize

At a Glance

PACING 1 day

Mathematical Goals (Objective)

- Use organized lists and tree diagrams to find theoretical probabilities
- Understand that experimental probabilities are better estimates of theoretical probabilities when based on larger numbers of trials

Launch

Use the opening paragraphs of Problem 2.3 to introduce tree diagrams. Demonstrate how to make a tree diagram and ask students questions modeling those they will need to ask themselves:

student groups will share tree diagrams before moving on to block Questions.

If I toss two coins,

- *What are the possible outcomes for the first coin?*
- *Are these outcomes equally likely?*
- *If you get heads with the first coin, what are the possible outcomes for the second coin? Are these outcomes equally likely?*
- *If you get tails with the first coin, what are the possible outcomes for the second coin?*
- *If you toss two coins, what is the probability that the coins will match? What is the probability that they won't match?*

Discuss the bonus game with students. Make sure that students realize that contestants must make a prediction before choosing a block from *each* bag. So they must say, for example, "blue from Bag 1 and red from Bag 2." Ask students to make a prediction:

- *What are the contestant's chances of winning this game?*

This activity works well either as a ~~whole-class experiment~~ or in groups, each group having two containers of blocks.

Materials

- Transparency 2.3
- Two opaque containers filled with 1 red, 1 yellow, and 1 blue block (if you choose to do this problem in groups, each group will need two containers)

Vocabulary

- tree diagram

Explore

Allow students to experiment and decide when they have enough data to make a good estimate of a contestant's chances of winning. If students do the activity in groups, pool the class's data before students answer Question B. Some may need help making a tree diagram. Have them ask themselves:

student groups will share sets of block combinations and strategies

- *What are the possible outcomes for the first choice? For the second choice?*
- *How do you know when you have all possible outcomes?*

Students may need support in determining which of the outcomes represent a win for the contestant.

Remind students that contestants are given one guess to correctly predict the color of block chosen from both bags.

Give chart paper or transparencies to each group to record their strategies for finding the theoretical probabilities.

Materials

- Chart paper or blank transparencies (optional)

some might list others might make tree diagrams, others tables

Reiterate that we combined our data because the more trials the more accurate the estimate of theoretical probabilities.

Summarize

- *What are the experimental probabilities for predicting each pair of colors?*
- *How did you determine the theoretical probabilities?*

Compare the theoretical probabilities to the experimental probabilities.

Have a class discussion about the kind of outcomes in Problem 2.1 and the kind of outcomes in this problem.

Materials
- Student notebooks

ACE Assignment Guide for Problem 2.3

Differentiated Instruction

Core 7–9
Other *Connections* 25–31; *Extensions* 35, 36; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 7 and other ACE exercises, see the CMP *Special Needs Handbook.*
Connecting to Prior Units 25–28: *Bits and Pieces I*; 29: *Data About Us*; 30, 31: *Bits and Pieces II*

Answers to Problem 2.3

A. **1.** Answers will vary but should be near the theoretical probability of $\frac{1}{9}$. This means each pair should be chosen approximately four times.

2. Answers will vary but should be near the theoretical probability of $\frac{1}{9}$. This means each pair should be chosen approximately four times.

3. In a larger set of data, the experimental probabilities should be closer to $\frac{1}{9}$.

B. **1.** (Figure 1) There are nine possible outcomes: RR, RY, RB, YR, YY, YB, BR, BY, and BB. Each pair is equally likely because there is exactly one of each color block in each of the bags and order matters in this game, so RY is different from YR.

2. The theoretical probability of each pair being chosen is $\frac{1}{9}$. Thus, the probability that a contestant will win the bonus prize is $\frac{1}{9}$.

3. Yes, each contestant has a $\frac{1}{9}$ chance of winning the bonus prize.

No, each contestant is more likely to lose than to win.

4. Twice. Because the probability of winning is $\frac{1}{9}$, you would expect to win once every nine times. Because $18 = 2 \times 9$, you would expect two wins.

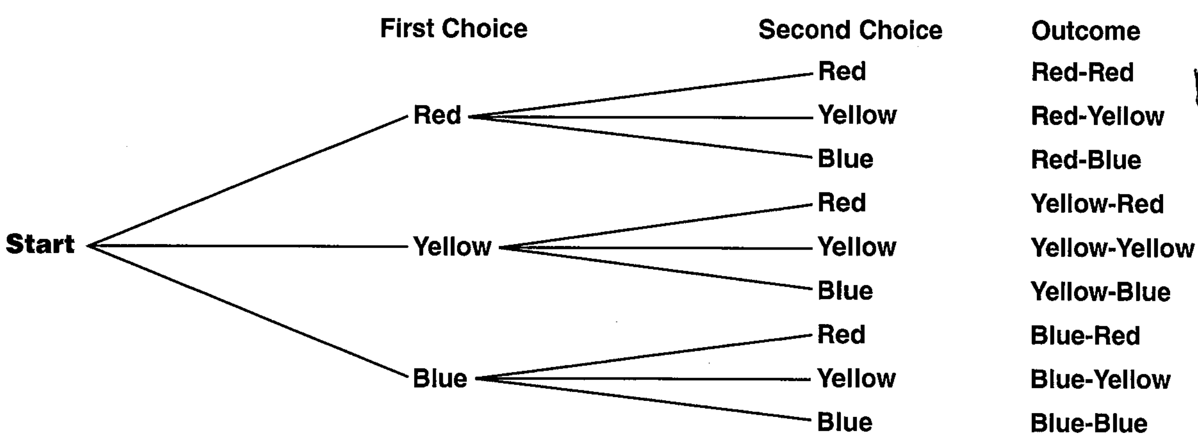
C. Answers will vary, but the two probabilities should be close.

or

	R	Y	B
R	RR	RY	RB
Y	YR	YY	YB
B	BR	BY	BB

Figure 1

Tree Diagram—Choosing Two Blocks



Closing Question/Check for Understanding: 10

Suppose Jason has 4 shirts and 3 pairs of pants. If he chooses a shirt and pants at random, how many different combinations can he make?

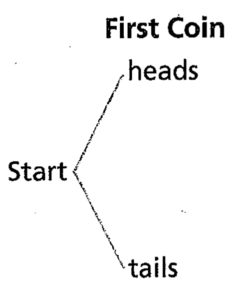
2.3 Winning the Bonus Prize

To find the theoretical probability of a result, you need to count all the possible outcomes. In some situations, such as when you toss a coin or roll a number cube, it is easy to count the outcomes. In other situations, it can be difficult. One way to find (or count) all the possible outcomes is to make an organized list. Here is an organized list of all the possible outcomes of tossing two coins.

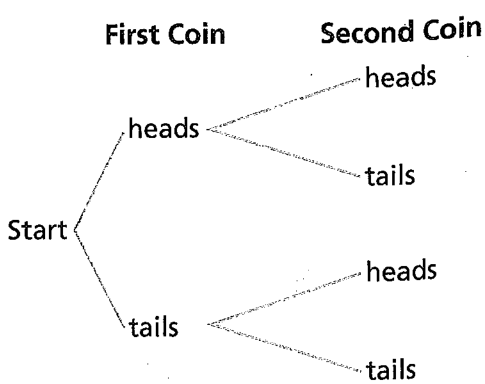
First Coin	Second Coin	Outcome
heads	heads	heads-heads
heads	tails	heads-tails
tails	heads	tails-heads
tails	tails	tails-tails

Another way to find all possible outcomes is to make a **tree diagram.** A tree diagram is a diagram that shows all the possible outcomes of an event. The steps for making a counting tree for tossing two coins are shown below.

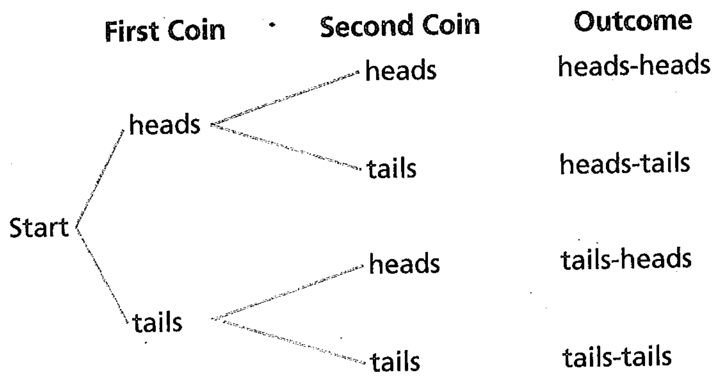
Step 1 Label a starting point. Make a branch from the starting point for each possible result for the first coin.



Step 2 Make a branch from each of the results for the first coin to show the possible results for the second coin.



Step 3 When you follow the paths from left to right, you can find all the possible outcomes of tossing two coins. For example, the path shown in red represents the outcome heads-heads.

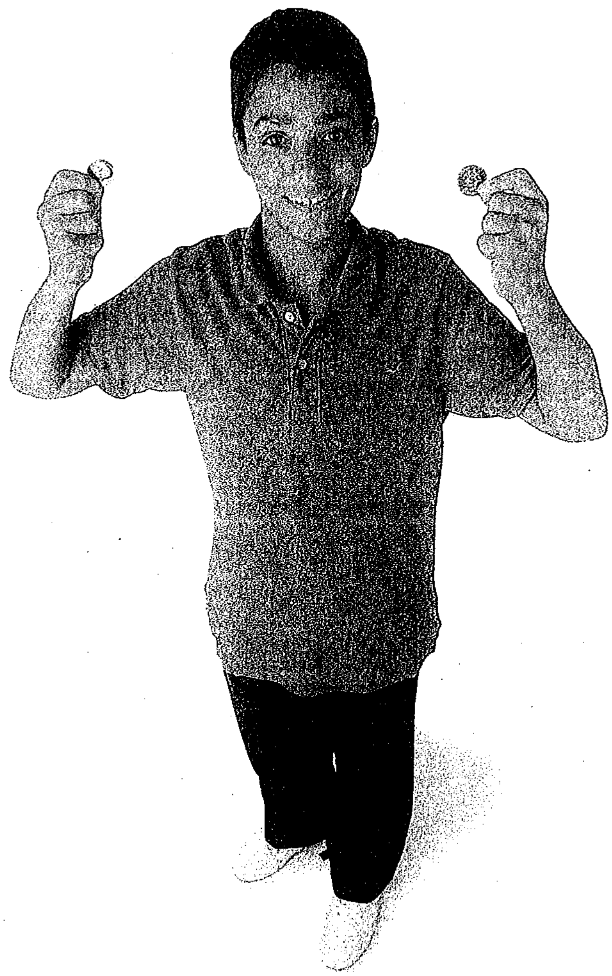


Both the organized list and the tree diagram show that there are four possible outcomes when you toss two coins. The outcomes are equally likely, so the probability of each outcome is $\frac{1}{4}$.

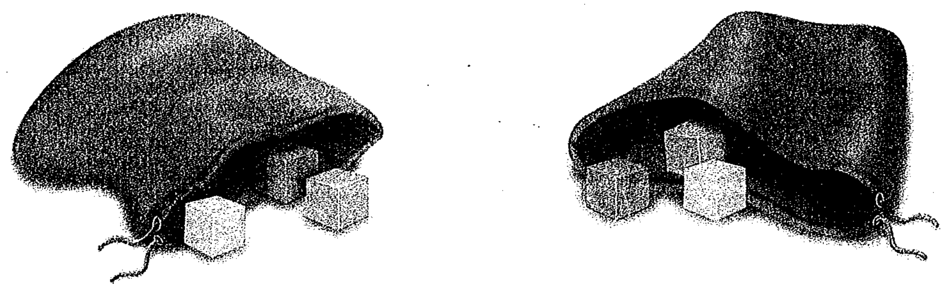
$$P(\text{heads, heads}) = \frac{1}{4}$$
$$P(\text{heads, tails}) = \frac{1}{4}$$
$$P(\text{tails, heads}) = \frac{1}{4}$$
$$P(\text{tails, tails}) = \frac{1}{4}$$

If you toss two coins, what is the probability that the coins will match?

What is the probability they won't match?



All the winners from the *Gee Whiz Everyone Wins!* game show have the opportunity to compete for a bonus prize. Each winner chooses one block from each of two bags. Both bags contain one red, one yellow, and one blue block. The contestant must predict which color she or he will choose from each of the two bags. If the prediction is correct, the contestant wins a $10,000 bonus prize!



What are the contestant's chances of winning this game?

Problem 2.3 Using Strategies to Find Theoretical Probabilities

A. **1.** Conduct an experiment with 36 trials for the situation above. Record the pairs of colors that you choose.

2. Find the experimental probability of choosing each possible pair of colors.

3. If you combined your data with the data collected by your classmates, would your answer to part (1) change? Explain.

B. **1.** List all the possible pairs that can be chosen. Are these outcomes equally likely? Explain your reasoning.

2. Find the theoretical probability of choosing each pair of blocks.

3. Does a contestant have a chance to win the bonus prize? Is it likely a contestant will win the bonus prize? Explain.

4. If you play this game 18 times, about how many times do you expect to win?

C. How do the theoretical probabilities compare with your experimental probabilities? Explain any differences.

ACE **Homework starts on page 28.**

1.1 Designing Bumper-Car Rides

At a Glance

PACING 1 day

Mathematical Goals

- Learn that the area of a figure is the number of square units needed to cover it
- Learn that the perimeter of an object is the number of units of length needed to surround it

Launch

Tell students about the bumper-car rides.

- *Let's think of this tile as the world's simplest bumper-car floor plan. A design that consists of only one tile represents a 1-m² design that would require 4 m of bumper rail to surround it.*

Begin a table on the board for recording data. Have students find the possible ways to arrange 2 tiles. Make sure the class understands that the tiles of bumper-car floor plans must fit together edge to edge. Discuss and record on the table. Repeat with 3 tiles.

- *Show me a design with 4 m² of flooring. How many meters of railing does it need?*

Ask students whether there are other numbers of tiles that can be arranged in more than one way so that different numbers of rail sections are required. Do not try to find a definite answer at this time. Leave the question for students to think about.

Have students make sketches of their designs on grid paper in pairs.

Materials

- Square tiles (24 per student or pair)
- Centimeter grid paper

Explore

Students will need time to talk about and experiment with the tiles. As you visit with them, be sure their sketches for Questions A and B are complete and clearly show 36 tiles have been used in each design.

If students are still struggling with constructing designs for a given number of tiles or figuring out how many rails are needed, continue by asking students to design bumper-car floors with 12 tiles.

Summarize

Have pairs share their results for Questions A and B. Ask students to explain why their designs meet the requirements. Keep students focused on the mathematics and what is happening to the perimeter (the number of rails needed) as they look at the variety of designs with the same area.

Talk about the labels for area and perimeter measure by relating each to what is being counted.

- *What are you actually counting when you measure area?*

Materials

- Transparency 1.1
- Student notebooks

Vocabulary

- area
- perimeter

continued on next page

Summarize *continued*

- *What are you actually counting when you measure perimeter?*
- *How are these measurements different from each other?*

Talk with students about the importance of labeling measures.

ACE Assignment Guide for Problem 1.1

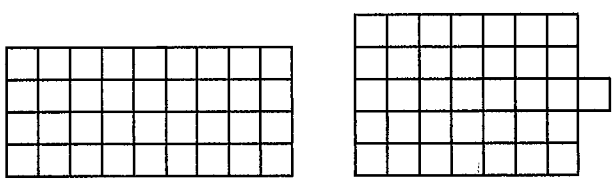
Differentiated Instruction Solutions for All Learners

Core 1–5
Other *Applications* 6

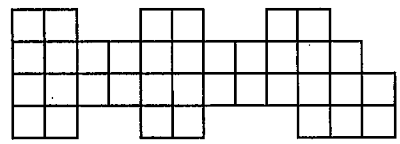
Adapted For suggestions about adapting Exercise 6 and other ACE exercises, see the CMP *Special Needs Handbook.*

Answers to Problem 1.1

A. Possible answers: The floor plan could be a 9-by-4 rectangle, or a 5-by-7 rectangle with an additional square somewhere.



B. Possible answer:



The area is 36 m^2. The perimeter is 40 m.

The largest perimeter possible for 36 squares and whole-number dimensions is a 1-by-36 rectangle with a perimeter of 74 m, but it wouldn't make a very interesting bumper-car design. NOTE: There are other, nonrectangular arrangements with this same perimeter.

C. 1. area: 45 m^2; perimeter: 32 m

2. Possible answer: Area is probably a better measure, because it indicates the amount of space available for the bumper cars to move around on.

* note alot of this lesson is working with the tiles and practicing area and perimeter.

Questions to ask during Explore or Share

Investigation 1

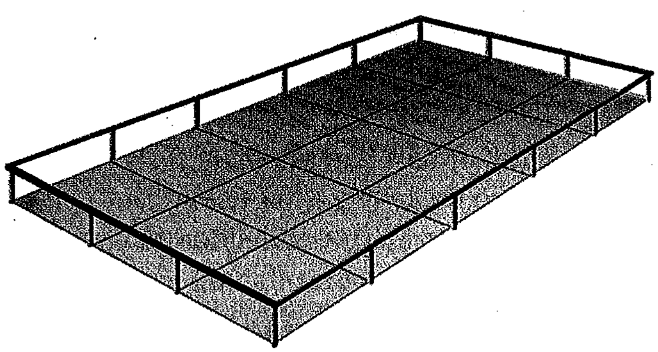
Designing Bumper Cars

Most people enjoy the rides at amusement parks and carnivals, from merry-go-rounds and Ferris wheels to roller coasters and bumper cars.

Suppose a company called Midway Amusement Rides (MARS for short) builds rides for amusement parks and carnivals. To do well in their business, MARS designers have to use mathematical thinking.

1.1 Designing Bumper-Car Rides

Bumper cars are a popular ride at amusement parks and carnivals. Bumper cars ride on a smooth floor with bumper rails all around it. MARS makes their bumper-car floors from 1 meter-by-1 meter square tiles. The bumper rails are built from 1-meter sections.



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Problem 1.1 Understanding Area and Perimeter

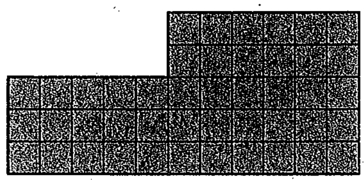
When a customer sends an order, the designers at MARS first use square tiles to model possible floor plans. MARS has received the customer orders below. Experiment with square tiles and then sketch some designs for the customer to consider.

A. Badger State Shows in Wisconsin requests a bumper-car ride with 36 square meters of floor space and 26 meters of rail sections. Sketch two or three floor plans for this request.

B. Lone Star Carnivals in Texas wants a bumper-car ride that covers 36 square meters of floor space and has lots of rail sections. Sketch two or three possible floor plans for this customer.

C. Two measures tell you important facts about the size of the bumper-car floor plans you have designed. The number of tiles needed to cover the floor is the **area.** The number of rail sections needed to surround the floor is the **perimeter.**

1. What are the area and perimeter of this bumper-car floor plan?



2. Which measure, perimeter or area, do you think better describes the *size* of a bumper-car floor plan? Why?

ACE Homework starts on page 10.

1.2 Pricing Bumper-Car Rides

When it is time to prepare the estimates or bills for customers, the designers at MARS turn over the plans to the billing department. The company charges $25 for each rail section and $80 for each floor tile.

1.2 Pricing Bumper-Car Rides

At a Glance

PACING 1 day

Mathematical Goals (Objectives)

- Learn that the area of a figure is the number of square units needed to cover it
- Learn that the perimeter of an object is the number of units of length needed to surround it
- Understand that two figures with the same area may have different perimeters

Launch

Let students strategize how to find cost.

Provide students with a copy of Labsheet 1.2. Read through the problem with your class. Revisit the definitions of area and perimeter. You might find it helpful to work as a class to fill in the table for Design A. Allow students to continue working on the problem, individually or in pairs.

after students come

Materials

- Square tiles (24 per student or pair)
- Labsheet 1.2

Explore

Help those students who are still confused about perimeter. Have some tiles ready for students who want to make the shapes themselves.

Look to see if students understand what Question B is asking. For part (2), see if students chose designs with a common area.

Question D asks students to design bumper-car floor plans. As students work, look for interesting floor plans. Ask these students to make an overhead transparency of their floor plan to share in the summary.

* As an extra challenge, you might ask some students to design two floor plans with the same cost, but different areas.

Materials

- Transparency 1.2

→ pass out tiles anyway, they don't have to use them but some want and refuse to ask.

Summarize

SHARE

Ask students to share the information in their charts. Encourage students to look for relationships between area and perimeter.

- *What is the area of Design A? How did you find it? What were you counting?*
- *What is the perimeter of Design A? How did you find it? What were you counting?*

Check that students have found area for Question B, part (1).

- *Which part of the table, area or perimeter, did you use to answer Question B, part (1)? Why?*

Discussion should emphasize that although many floor plans have the same area, the perimeter and cost vary.

- *Do any other designs have an area of 16 m^2? Do all designs with 16 m^2 of floor space cost the same?*

Ask students to explain what causes designs with the same area and different perimeters to have different costs.

Be sure to ask students to explain their reasoning in Question C.

Materials

- Student notebooks

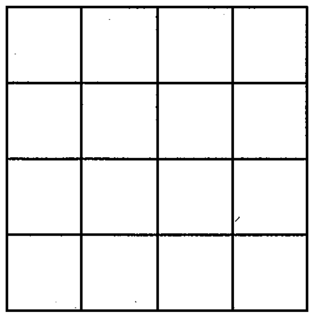
continued on next page

Name ______________________ Date ____________ Class ____________

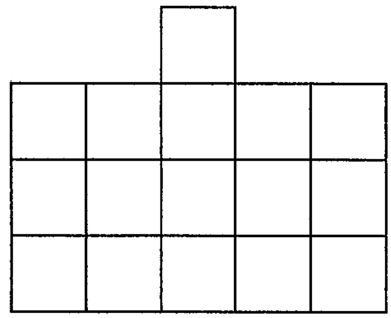
Labsheet 1.2

Designs A–H

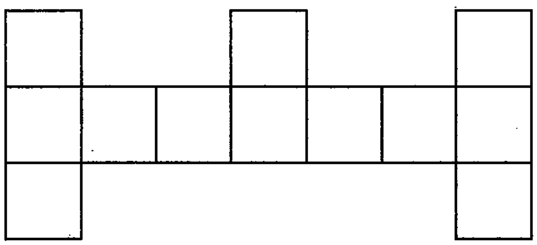
Design A



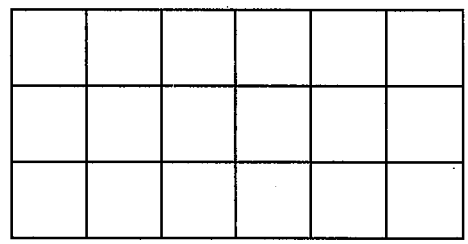
Design B



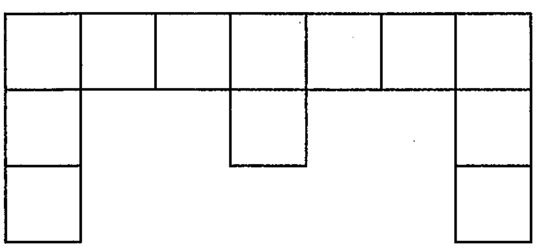
Design C



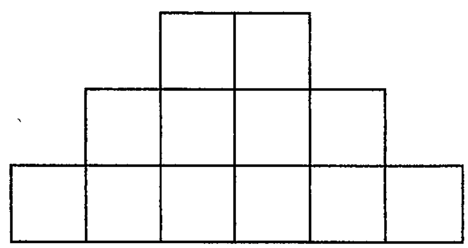
Design D



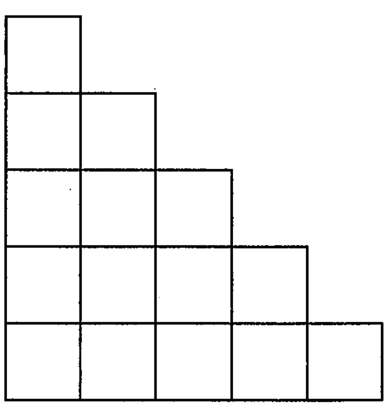
Design E



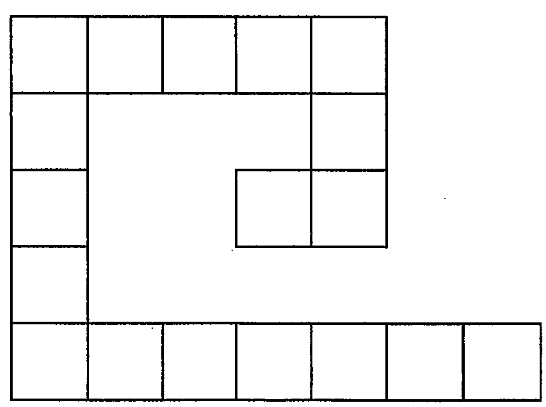
Design F



Design G



Design H



Problem 1.1 Understanding Area and Perimeter

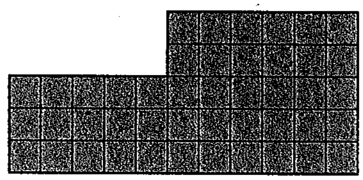
When a customer sends an order, the designers at MARS first use square tiles to model possible floor plans. MARS has received the customer orders below. Experiment with square tiles and then sketch some designs for the customer to consider.

A. Badger State Shows in Wisconsin requests a bumper-car ride with 36 square meters of floor space and 26 meters of rail sections. Sketch two or three floor plans for this request.

B. Lone Star Carnivals in Texas wants a bumper-car ride that covers 36 square meters of floor space and has lots of rail sections. Sketch two or three possible floor plans for this customer.

C. Two measures tell you important facts about the size of the bumper-car floor plans you have designed. The number of tiles needed to cover the floor is the **area.** The number of rail sections needed to surround the floor is the **perimeter.**

1. What are the area and perimeter of this bumper-car floor plan?



2. Which measure, perimeter or area, do you think better describes the *size* of a bumper-car floor plan? Why?

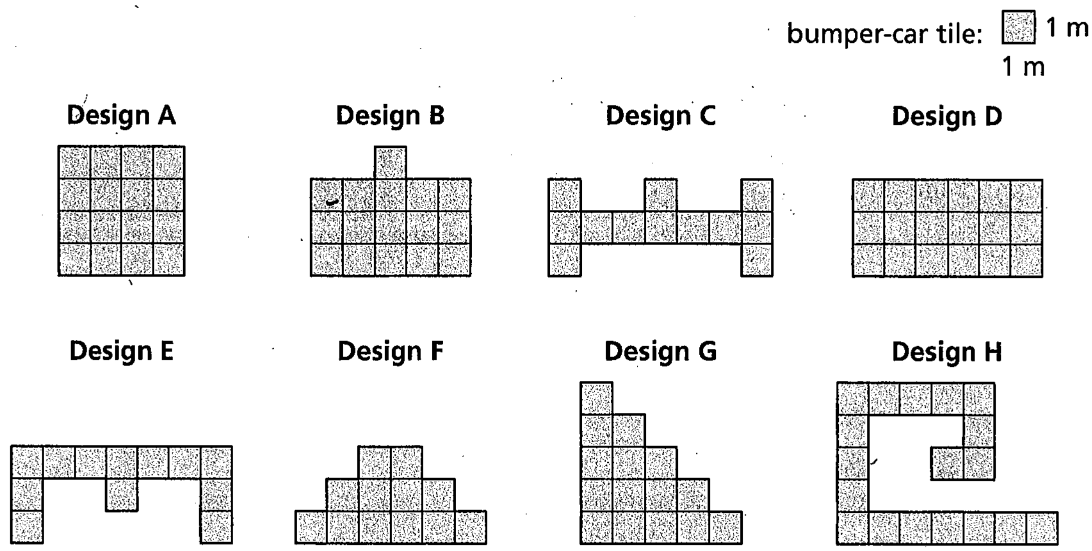
ACE Homework starts on page 10.

1.2 Pricing Bumper-Car Rides

When it is time to prepare the estimates or bills for customers, the designers at MARS turn over the plans to the billing department. The company charges $25 for each rail section and $30 for each floor tile.

Problem 1.2 Finding Area and Perimeter

The Buckeye Amusement Company in Ohio wants some sample floor plans and cost estimates for bumper-car rides. The designers come up with these bumper-car floor plans.



A. Find the area and perimeter for each design. Record your data in a table such as the one started at the right.

Design	Area	Perimeter	Cost
A			
B			

B. Use the data in your table.

1. Which designs can be made from the same number of floor tiles?

2. Choose a set of designs that can be made from the same number of floor tiles. What is the perimeter of each design?

3. In the designs with the same floor area, which design costs the most? Which design costs the least? Why?

C. 1. Rearrange the tiles in Design H to form a rectangle. Can you make more than one rectangle using the same number of tiles? If so, are the perimeters of the rectangles the same? Explain.

2. Design B and Design D have the same perimeter. Can you rearrange Design B to make Design D? Explain.

D. 1. The Buckeye Amusement Company said that it is willing to pay between $1,000 and $2,000 for a bumper-car ride. Design two possible floor plans. Find the area, perimeter, and cost for each.

2. Suppose you were the manager. Which design would you choose? Why?

ACE **Homework starts on page 10.**

1.3 Decoding Designs

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goal (Objectives)

- Use the relationship between length and width to develop formulas for the area and perimeter of a rectangle

Answer the getting ready in Think-Pair-Share
help them see that the student would get the same perimeter measurement if she counted the exposed side lengths.

Launch

Discuss the Getting Ready with students.

Give each student a copy of Labsheet 1.3. Read through the problem with the students. Be sure they understand that Questions A, B, and C ask them to consider bumper-car floor plans that come with different types of diagrams and descriptions: gridded diagrams with dimensions labeled, and written descriptions.

Introduce the term *dimension.*

Ask students to think about Problem 1.2, Question C, where they were asked to make rectangular bumper-car floor plans using 18 floor tiles.

- *Tell me the dimensions or the length and width of a rectangle you can make with 18 floor tiles.*

Have students work in groups ~~of 2 or 3.~~ table

Materials
- Transparency 1.3A
- Labsheet 1.3
- String: one piece at least 18 cm long per student (optional)

Explore

As students work, look at the different explanations students write for Question A, parts (1) and (2).

- What strategies are students using to find area and perimeter?
- What are students counting when they find area?
- What are students counting when they find the perimeter?
- Do students have other strategies besides counting for finding area and perimeter?
- Do they use the terms *length* and *width* in conversation?

If students have trouble with Question C, suggest they draw a diagram of the bumper-car floor plan and label the length and width.

Materials
- Transparency 1.3B

for B students can sketch in lines or use tiles. & same with IV and V.

Summarize

SHARE

Ask students to describe their strategies for finding area and perimeter in words. Discuss area first, then perimeter rather than proceeding through the parts of the problem in order.

Write students' ideas on the board according to what students say. For example:

Mari: There are 6 squares in a row and 6 rows, so I multiplied 6 × 6 to get 36.

Materials
- Transparency 1.3C
- Student notebooks

Vocabulary
- dimension

continued on next page

* Another student might have counted by 6's or counted individual squares

Summarize *continued*

On the board, you might write:

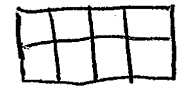
number of squares in a row × number of rows = area
6 squares × 6 = 36 squares

Erik: I just multiplied the length and the width.

You might ask how Erik's idea of *length* × *width* is similar to Mari's idea.

~~Question D leads to the introduction of the area formula $A = \ell \times w$. Relate this formula back to Questions A–C or with some problems you offer.~~

Repeat this sequence with perimeter: have students share their strategies while you record and push towards a formula.



"what are the dimensions"
4 by 2
"what does the 4 tell us?"
4 cm² in a row
"what does the 2 tell us?"
there are 2 rows of 4

ACE Assignment Guide for Problem 1.3

Differentiated Instruction Solutions for All Learners

Core 16–21, 31

Other *Applications* 22–27; *Connections* 32, 33, 36–38; *Extensions* 41, 42; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook.*

Connecting to Prior Units 31: *Prime Time*

Answers to Problem 1.3

A. 1. Design I: 36 m^2
Design II: 40 m^2
Design III: 55 m^2
Design IV: 54 m^2
Design V: 77 m^2

Strategies for finding area may vary. Any reasonable strategy, including counting the number of squares, is acceptable.

2. Design I: 24 m
Design II: 26 m
Design III: 32 m
Design IV: 30 m
Design V: 36 m

Strategies for finding perimeter may vary. Any reasonable strategy is acceptable.

B. area: 48 m^2; perimeter: 32 m

C. 1. area: 510 m^2

2. perimeter: 94 m

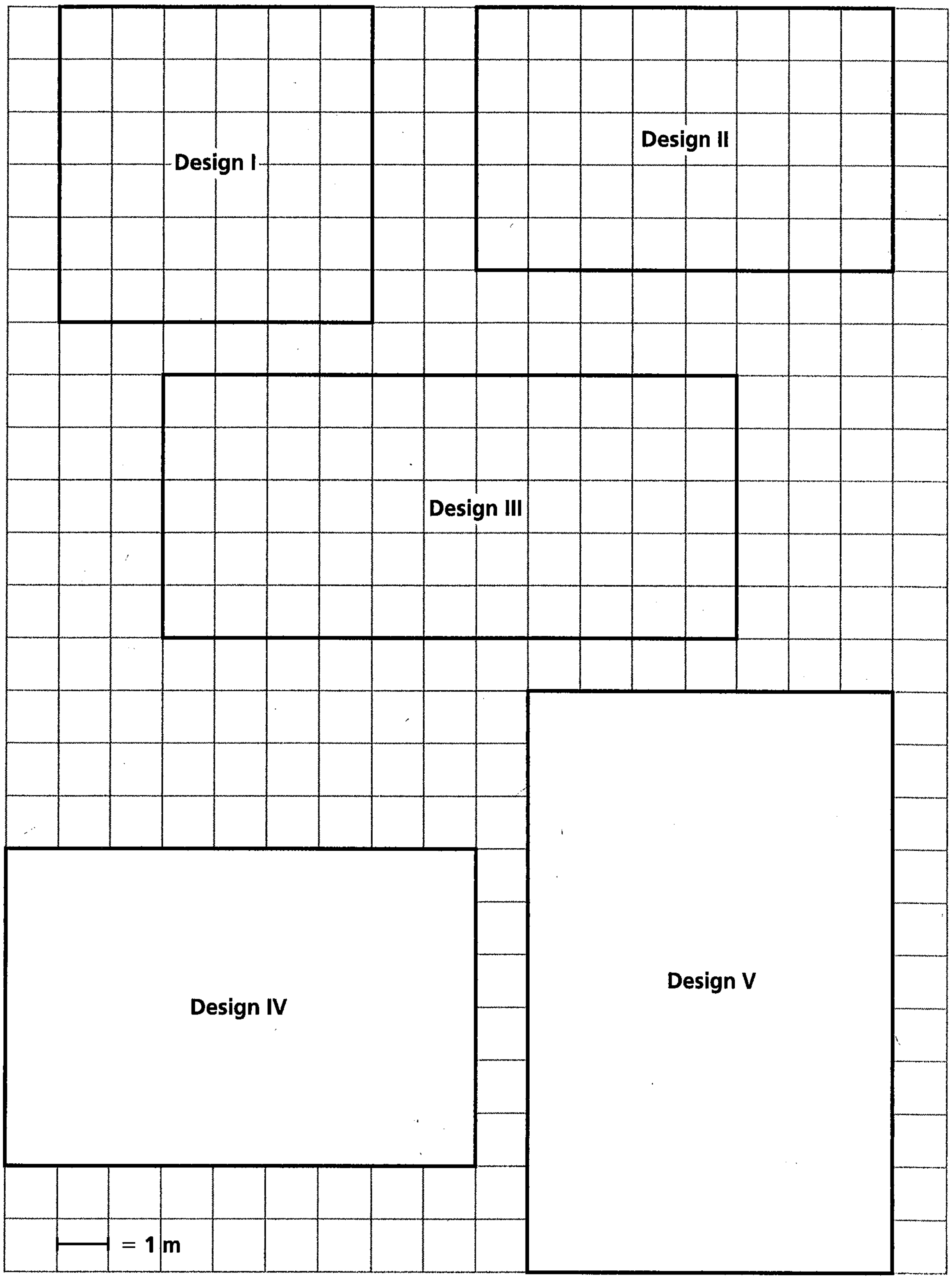
D. 1. Possible answers include:
$P = \ell + w + \ell + w$, or
$P = (\ell + w) \times 2$, or
$P = 2 \times \ell + 2 \times w$

2. Area $= \ell \times w$ or $A = \ell \times w$

Name ______________________ Date __________ Class __________

Labsheet 1.3

Designs I–V

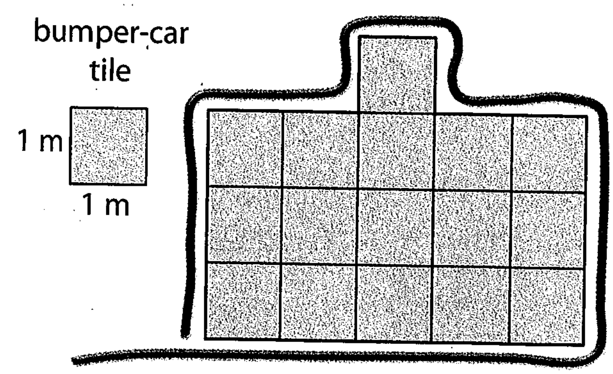


24

Getting Ready for Problem 1.3

A student is tired of counting the individual rail sections around the outside of each bumper-car track. She starts to think of them as one long rail. She wraps a string around the outside of Design B, as shown.

What do you think she does next? How does this help her to find the perimeter of the figure? How could she determine the area?



1.3 Decoding Designs

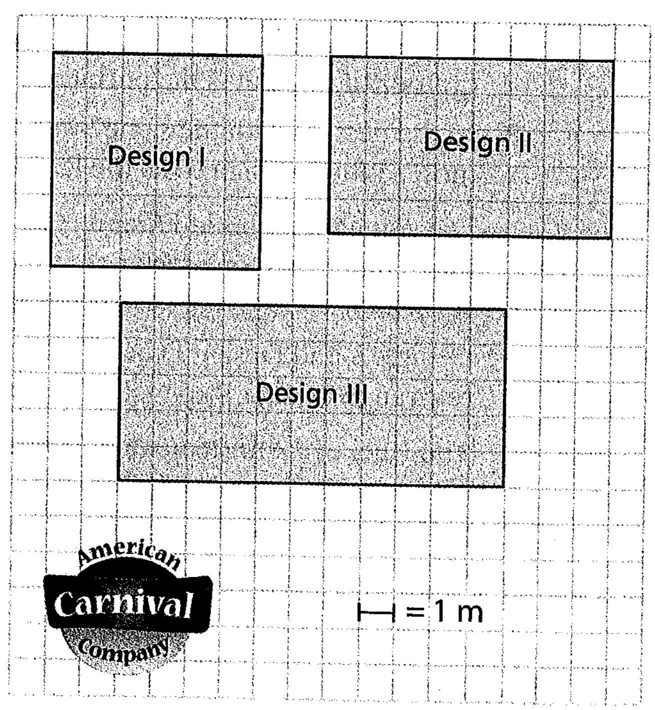
The Portland Community Events Council is planning its annual summer festival. The council asks for bids from different traveling carnival shows. Each carnival show sends descriptions of the rides they offer.

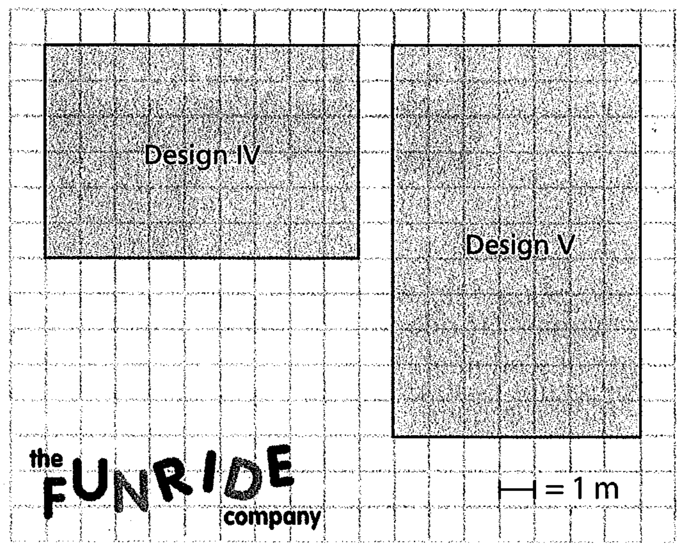
Problem 1.3 Finding Area and Perimeter of Rectangles

The council wants to have a bumper-car ride in the shape of a rectangle at the festival.

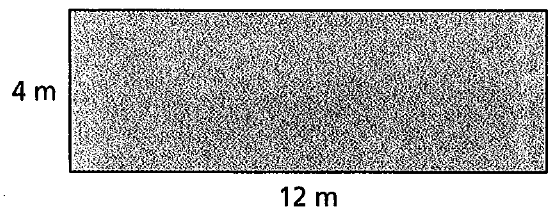
A. American Carnival Company sends Designs I, II and III. The Fun Ride Company sends Designs IV and V (on the next page).

1. What is the area of each design? Explain how you found the area.
2. What is the perimeter of each design? Explain how you found the perimeter.





B. One carnival company sends the rectangular floor plan below. Find the area and the perimeter of this floor plan.



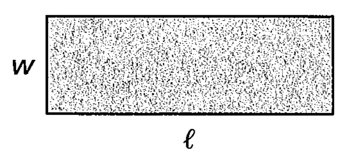
C. Another carnival company sends a description rather than a diagram. They describe the ride as a rectangle that is 17 meters by 30 meters.

1. What is the area of this floor plan?

2. What is the perimeter of this floor plan?

D. The dimensions of a rectangle are called **length** and **width.** Length can be represented using ℓ and width can be represented using w.

1. Using ℓ for length and w for width, write a rule for finding the perimeter of a rectangle.



2. Using ℓ for length and w for width, write a rule for finding the area of a rectangle.

ACE **Homework starts on page 10.**

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At a Glance

2.1 Building Storm Shelters

PACING $1\frac{1}{2}$ days

Mathematical Goals (Objectives)

- Explore questions of maximum and minimum in the context of finding the largest and smallest perimeter for rectangles of fixed area
- Understand that the perimeters of rectangles with a fixed area can vary considerably
- Continue to develop facility-using formulas for finding perimeter and area of rectangles
- Continue to develop a conceptual understanding of area and perimeter

Launch

*use tiles!

Help students understand the mathematical context.

Ask students to use 12 of their tiles to build a rectangle.

- *We call the length and width of a rectangle its dimensions. Tell me what rectangle you built by giving me its dimensions.*

Record the length, width, perimeter, and area of each rectangle on the board. Sketch the rectangles students describe. Relate their work on this problem to the rules for area and perimeter of rectangles. Collect students' ideas until all possible rectangles have been found.

- *How do you know we have found all the rectangles that can be made using 12 tiles?* - talk about factor pairs

SHARE

Read about building storm shelters with your class.

- *As you work on this problem, record your findings in a table similar to the one we used to find the rectangle with an area of 12 square units.*

Have students work in pairs.

Materials

- Square tiles (24 per student or pair)
- Labsheet 2.1
- Transparencies 2.1A, 2.1B

Explore

Have students share strategies then tables and lastly graphs on the document camera.

Make sure students make sketches and complete the chart. Help students plot points on their graphs when necessary. Help students understand that each point gives two pieces of information.

Summarize

Collect the data students recorded for Question A.

- *Did anyone find a shelter design with an edge length of 1 m? What is the width of that shelter? The perimeter? How did you find the perimeter?*

Continue with this line of questioning for edge lengths of 3, 4, 5, 6, 7, 8, 12, and 24.

SHARE

- *Which of the shelters with an area of 24 m^2 has the smallest perimeter? What does it look like? What is the perimeter of this design?*
- *Which of the shelters with an area of 24 m^2 has the largest perimeter? What does this shelter look like?*
- *Why do some floor plans use more wall panels than others?*

Materials

- Student notebooks

Vocabulary

- fixed area
- maximum perimeter
- minimum perimeter

continued on next page

"What does the 4x6 look like?" (square-like)
"What does the 1x24 look like? (long and skinny)

Summarize
continued

Go over the graph and its meaning with students.

- *We said that the design with the smallest perimeter is 4 m by 6 m. How should we display this on the graph?*

Have students state a general rule about maximizing and minimizing perimeters for rectangles with fixed area. based on the PATTERNS.

important!

ACE Assignment Guide for Problem 2.1

Differentiated Instruction Solutions for All Learners

Core 1, 2

Other *Applications* 3–6; *Connections* 16, 17, 20, 21 Labsheet 2ACE Exercises 3–5 is provided if Exercises 3–5 are assigned.

Adapted For suggestions about adapting Exercise 3 and other ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 17, 20, 21: *Bits and Pieces II*

Answers to Problem 2.1

A.

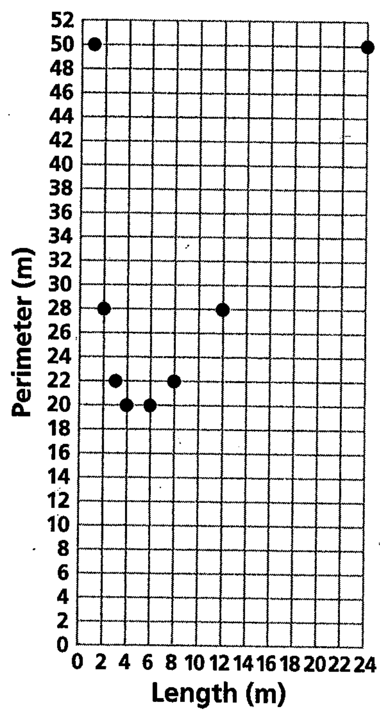
Length (m)	Width (m)	Perimeter (m)	Area (m²)
1	24	50	24
2	12	28	24
3	8	22	24
4	6	20	24
6	4	20	24
8	3	22	24
12	2	28	24
24	1	50	24

B. 1. Perimeter; possible explanation: Perimeter is the distance or length around the outside of a shape. The walls fit around the outside so the number of panels depends on how long the distance around the outside is.

2. The 1 m-by-24 m (or 24 m-by-1 m) shelter is the most expensive to build. The floor plan is long and skinny, with the least open space and the most wall sections.

3. The 4 m-by-6 m (or 6 m-by-4 m) shelter is the least expensive to build. The floor plan is the most square-like of the possibilities. The floor plan would have the most open space and the fewest panels.

C. 1.



2. The graph is curved. As we move from left to right, the points are lower and lower, then begin to rise. This is because if one side of the storm shelter is very short, the other is very long and the perimeter will be large. As the two dimensions become close to each other, the perimeter becomes smaller. Past a certain point, the perimeter becomes large again.

D. 1. The 6 m-by-6 m floor plan would have the least perimeter and the 36 m-by-1 m floor plan would have the greatest perimeter.

2. A long, skinny rectangle has the largest perimeter for a fixed area, while the rectangle that is most square-like has the smallest perimeter for a fixed area.

Investigation 2

Changing Area, Changing Perimeter

Whether you make a floor plan for a bumper-car ride or a house, there are many options.

You should consider the cost of materials and the use of a space to find the best possible plan. In Investigation 1, you saw that floor plans with the same area could have different perimeters. Sometimes you want the largest, or *maximum*, possible area or perimeter. At other times, you want the smallest, or *minimum*, area or perimeter.

This investigation explores these two kinds of problems. You will find the maximum and minimum perimeter for a fixed area. You will also find the maximum and minimum area for a fixed perimeter. *Fixed* area or perimeter means that the measurement is given and does not change.

2.1 Building Storm Shelters

Sometimes, during a fierce winter storm, people are stranded in the snow, far from shelter. To prepare for this kind of emergency, parks often provide shelters at points along major hiking trails. Because the shelters are only for emergency use, they are designed to be simple buildings that are easy to maintain.

Problem 2.1 Constant Area, Changing Perimeter

The rangers in a national park want to build several storm shelters. The shelters must have 24 square meters of rectangular floor space.

A. Experiment with different rectangles that have whole-number dimensions. Sketch each possible floor plan on grid paper. Record your data in a table such as the one started below. Look for patterns in the data.

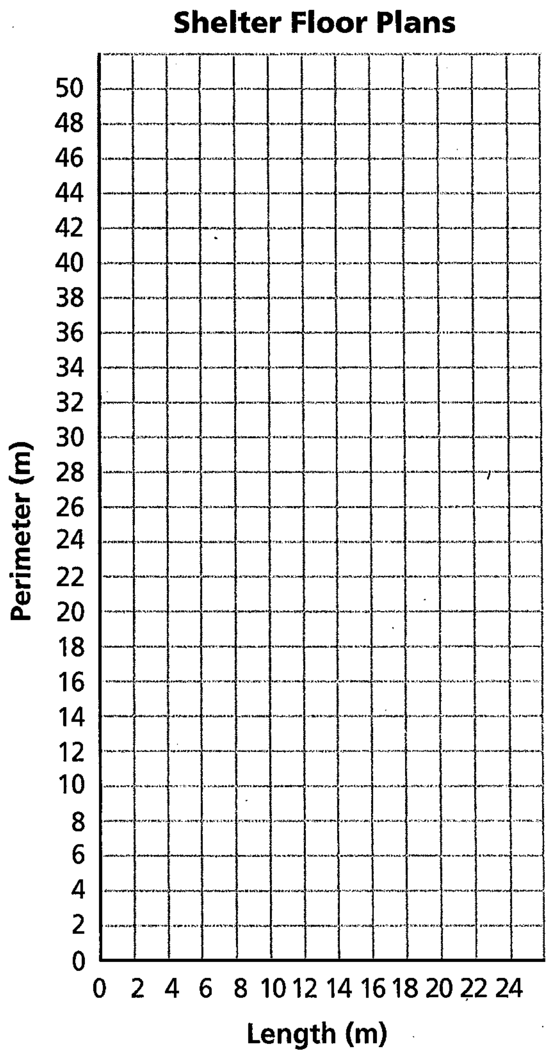
Shelter Floor Plans

Length	Width	Perimeter	Area
1 m	24 m	50 m	24 sq. m

B. Suppose the walls are made of flat rectangular panels that are 1 meter wide and have the needed height.

1. What determines how many wall panels are needed, area or perimeter? Explain.
2. Which design would require the most panels? Explain.
3. Which design would require the fewest panels? Explain.

C. 1. Use axes like the ones below to make a graph for various rectangles with an area of 24 square meters.



2. Describe the graph. How do the patterns that you observed in your table show up in the graph?

D. 1. Suppose you consider a rectangular floor space of 36 square meters with whole-number side lengths. Which design has the least perimeter? Which has the greatest perimeter? Explain your reasoning.

2. In general, describe the rectangle with whole-number dimensions that has the greatest perimeter for a fixed area. Which rectangle has the least perimeter for a fixed area?

ACE **Homework starts on page 26.**

2.4 Adding Tiles to Pentominos

PACING 1 day

Mathematical Goals (Objectives)

- Distinguish the case of fixed area from fixed perimeter
- Apply understanding of the relationship between area and perimeter to nonrectangular figures
- Continue to develop a conceptual understanding of area and perimeter

SHARE / (Have students share using doc/cam or magnetic tiles.)

Launch

use magnetic tiles

Draw the pentomino on the board and have students construct it with their tiles.

- *What is the area of this figure? What is the perimeter?*

Tell students they must keep these five tiles in this arrangement. Have students record the area of each figure beside the figure they sketch on their labsheet.

Read through the problem with the students. Make students aware of what Questions B and C are asking so that they can think about these ideas as they work on Question A.

Have students work in pairs, but record their work individually.

Materials
- Square tiles (about 25 per student)

Stress that this involves experimenting — don't feel need to get answers from friends immediately

Explore

If students struggle with Questions B and C, ask them to experiment with how the perimeter of the figure changes when they add or subtract a tile in different places such as a corner or an edge.

Summarize

Help students to understand by demonstrating with diagrams or tiles.

- *When you add a tile it increases the area by one square unit. Will the perimeter increase, too?*
- *If you remove a tile and decrease the area, will the perimeter decrease, too? Why or why not?*

Discuss Questions B and C.

- *What is the fewest number of tiles you can add to the pentomino to get a perimeter of 18 units?*

Have students show different arrangements. There are several possibilities.

- *Where should you add the three tiles to get the perimeter to increase as quickly as possible?*
- *What is the greatest number of tiles you can add to get a perimeter of 18 units?*
- *What does the figure you got with the maximum area and a perimeter of 18 units look like?*

Materials
- Student notebooks

ACE Assignment Guide for Problem 2.4

Differentiated Instruction
Solutions for All Learners

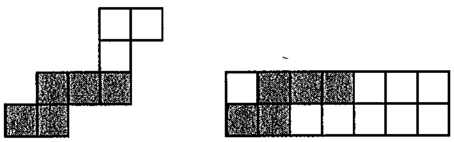
Core 15, 26

Other *Extensions* 28; unassigned choices from previous problems

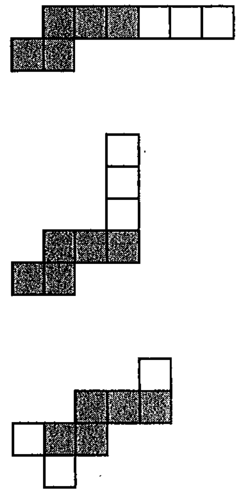
Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*

Answers to Problem 2.4

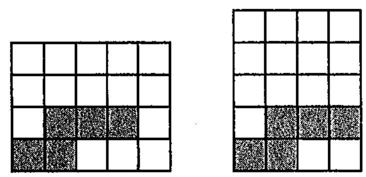
A. Answers will vary. Possible answers:



B. 3; possible answers (each of the three tiles must touch only one edge as they are added):



C. 15; possible answers (each figure must enclose the pentomino in a 4-by-5 rectangle)



MODEL THESE!!!

SUMMARIZE

Three Ideas about adding tiles should emerge

1) You can add a tile so it touches one end of a figure. (Adds 3 exposed edges, and eliminates 1, increasing perimeter by 2)

2) You can add a tile in the corner so that it touches 2 tiles. (Adds 2 exposed edges and eliminates 2 so perimeter is unchanged)

3) You can slip a tile into a space surrounded by 3 tiles. (Adds 1 exposed edge and eliminates 3, decreasing perimeter by 2)

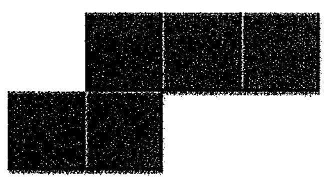
2.4 Adding Tiles to Pentominos

Shapes that are not rectangles can also be made from tiles. A *pentomino* (pen TAWM in oh) is a shape made from five identical square tiles connected along their edges. Turning or flipping a pentomino does not make a different pentomino, so these two figures are considered the same.

In this problem, you will add tiles to a pentomino and examine its area and perimeter.

Problem 2.4 Increasing Area and Perimeter

Make this pentomino with your tiles:



A. Add tiles to the pentomino to make a new figure with a perimeter of 18 units. Draw the new figure on grid paper. Show where you added tiles to the pentomino.

B. What is the fewest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? Draw the new figure, showing where you would add tiles to the pentomino.

C. What is the greatest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? Draw the new figure, showing where you would add tiles to the pentomino.

ACE **Homework starts on page 26.**

#use cubes!

1.1 Making Cubic Boxes

At a Glance

PACING 1 day

Mathematical Goals (objectives)

- Visualize a net as a representation of the surface area of a cube
- Connect the area of the net to the surface area of a cube

Launch

packages are often designed under a set of constraints. (save $, shape, material, space, etc..)

Discuss the work of packaging engineers. To help students begin thinking about packaging items, discuss with them the introduction to the investigation in the student edition. Discuss the idea that some boxes are cubes. Have students describe a cube.

- *What does a cube look like?*
- *What features of a cube could we count? We call the corners* vertices.
- *How many vertices does a cube have? How many edges does a cube have? We call the sides* faces. *How many faces does a cube have?*

Introduce the term *unit cube*. Review the special features that describe plane figures, such as dimensions, area, and perimeter. Corresponding measures will be developed for three-dimensional figures.

Cut out their nets to use during the share!

Be sure students understand that the cube cannot have two overlapping squares from the net.

Each student should make at least one new net. Students can work in groups of 2–3 to share the work of finding all of the nets. (table groups)

Materials
- Inch grid paper
- Inch cubes
- Transparencies 1.1A and 1.1B (optional)

Vocabulary
- Unit cube

Explore

As you circulate, ask students questions about the nets they are creating.

- *How do you know your nets will work? How could you show someone else that they will work?*
- *What things are the same in all of the nets? What things are different? How is the area of the net related to the number of squares that would be needed to cover the cube?*

Give some groups transparent grid paper to record their nets. These can be used in the summary. also the big cubes to a few students to model to the class

Materials
- Scissors

Summarize

SHARE

Ask students to display the various nets on the board or overhead projector. Discuss the nets that the class generated. Repeat the questions asked in the preceding Explore section.

- *What is the total area needed to cover a unit cube?*

You may want to use ACE Exercise 32 as an in-class wrap-up problem.

Materials
- Student notebook

ACE Assignment Guide for Problem 1.1

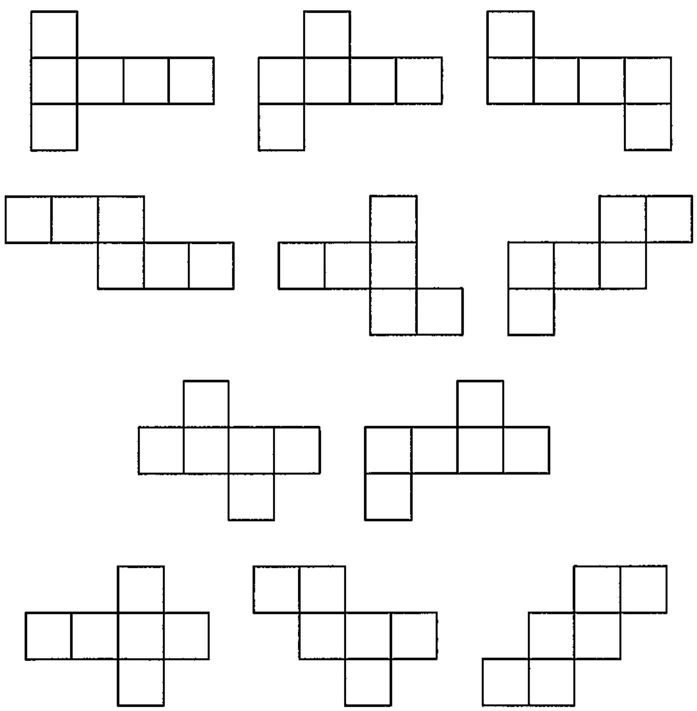
Differentiated Instruction
Solutions for All Learners

Core 1–3, 16
Other *Applications* 4, *Connections* 15, 17–18; *Extensions* 32

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 15, 18: *Covering and Surrounding*

Answers to Problem 1.1

A. There are 35 different nets that can be made with six squares (these are called hexominos). However, only the 11 shown below will fold into a cubic box. (These are shown on Transparency 1.1B.)



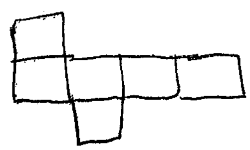
Discuss area in a net and surface area of a cube.

Clear up misconceptions if needed ->

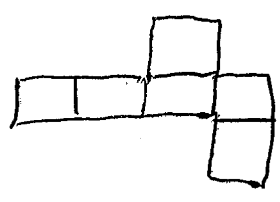
B. The area of each net is 6 square units. A unit cube has 6 faces, each of which has an area of 1 square unit.

SUMMARIZE

Some students may argue some nets are the same (others will disagree) talk about symmetry, rotations flips. For example



is the same shape/net as

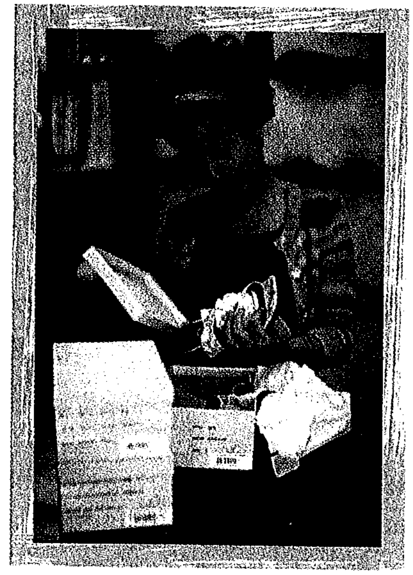


Closing? Think-Pair-Share "What is a net and why might they be helpful?"

Investigation 1

Building Boxes

The most common type of package is the rectangular box. Rectangular boxes contain everything from cereal to shoes and from pizza to paper clips. Most rectangular boxes begin as flat sheets of cardboard, which are cut and then folded into a box shape.



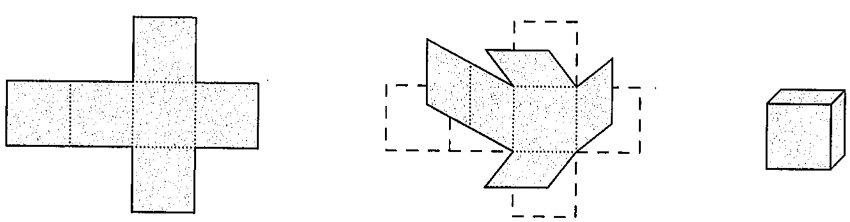
1.1 Making Cubic Boxes

Some boxes are shaped like cubes. A **cube** is a three-dimensional shape with six identical square faces.

What kinds of things might be packaged in cubic boxes?

The boxes you will work with in this problem are shaped like unit cubes. A **unit cube** is a cube with edges that are 1 unit long. For example, cubes that are 1 inch on each edge are called inch cubes. Cubes that are 1 centimeter on each edge are called centimeter cubes.

In this problem, you will make nets that can be folded to form boxes. A **net** is a two-dimensional pattern that can be folded to form a three-dimensional figure. The diagram below shows one possible net for a cubic box.



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Problem 1.1 Making Cubic Boxes

On grid paper, draw nets that can be folded to make a unit cube.

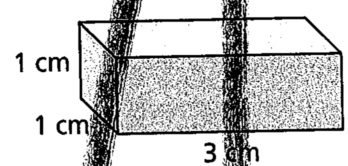
A. How many different nets can you make that will fold into a box shaped like a unit cube?

B. What is the total area of each net, in square units?

ACE Homework starts on page 10.

1.2 Making Rectangular Boxes

Many boxes are not shaped like cubes. The rectangular box below has square ends, but the remaining faces are non-square rectangles.



Problem 1.2 Making Rectangular Boxes

A. On grid paper, draw two different nets for the rectangular box above. Cut each pattern out and fold it into a box.

B. Describe the faces of the box formed from each net you made. What are the dimensions of each face?

C. Find the total area of each net you made in Question A.

D. How many centimeter cubes will fit into the box formed from each net you made? Explain your reasoning.

E. Suppose you stand the rectangular 1 centimeter × 1 centimeter × 3 centimeters box on its end. Does the area of a net for the box or the number of cubes needed to fill the box change?

ACE Homework starts on page 10.

active math online
For: Virtual Box Activity
Visit: PHSchool.com
Web Code: and-6102

1.2 Making Rectangular Boxes

PACING 1 day

Mathematical Goals (Objectives)

- Visualize a net as a representation of the surface area of a rectangular prism
- Connect the area of the net to the surface area of a rectangular prism

Launch

Hold up a rectangular box that is not a cube and ask students to describe it. Discuss the features of the box.

- *Describe the faces of this rectangular box. How many faces are there?*
- *How many edges does the box have?*
- *How many vertices does it have?*
- *Will a different box have a different number of faces, edges, or vertices?*

Hold up a different box.

Explain that packaging engineers may design a rectangular box by drawing a net that can be cut out and folded to make the box. Explain the challenge for students.

Have students work on their own, then compare results in pairs or threes.

Materials

- cm grid paper
- cm cubes (optional)
- Scissors
- 2 or 3 rectangular boxes

Vocabulary

- Dimensions

Explore

Guide stuck students to count the squares to help find measure.

Once students have drawn two nets and answered the questions about them, they should gather in their groups to compare their nets and validate that each could be folded into the same rectangular box.

Continue to ask questions like those you asked in Problem 1.1. Ask students to show that their nets will work; to look for things that are the same in all of the nets; to look for things that are different; and to consider how the area of each net is related to the number of squares that would cover the rectangular box.

"How did you find the area?" "What do you think the total area of the box's surface will be?"

Summarize

SHARE

Give students a chance to share and display their nets. Ask students how they found the area of their nets. The relationship between the area of the net and the surface area of the related box should be a focus of the discussion.

- *What was the area of your net?*
- *How did you find that measure?*
- *What do you think the total area of the box's surface will be? Why?*
- *How do these areas compare?*
- *Why does it make sense that these two measures are the same?*

Materials

- Student notebook

continued on next page

Make sure class realizes that cubes & rect angular
prisms have 6 faces, 12 edges, and 8 vertices.

Summarize

continued

Help students make the connection between the box's surface area and the area of plane figures.

- *What strategies did you use to find the number of unit cubes needed to fill the box?*

Demonstrate this filling. Use three cubes to demonstrate how they fill the rectangular boxes. Introduce the *dimensions* of a rectangular box: length, width, and height. Make sure students are aware that placing the box on a different face changes the base.

Give students the dimensions of a new box and ask them to sketch each face, labeling the dimensions and area of each face.

ACE Assignment Guide for Problem 1.2

Differentiated Instruction Solutions for All Learners

Core 5–7

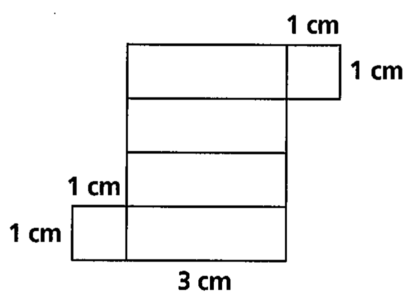
Other *Connections* 19–24; unassigned choices from previous problems

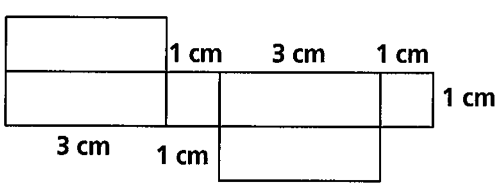
Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 19–22: *Covering and Surrounding*

Answers to Problem 1.2

A. Possible nets:





B. Four rectangular faces are congruent, with a length of 3 cm and a width of 1 cm. The remaining two faces are also congruent, with a length of 1 cm and a width of 1 cm.

C. The area for each net in Question A is 14 cm^2.

D. 3 cm cubes. One cube will cover the square end and fill one third of the box, so two more will fill the box.

E. No. If the position of the box is changed, the area of a net for the box and number of cubes needed to fill the box remain the same.

Problem 1.1 Making Cubic Boxes

On grid paper, draw nets that can be folded to make a unit cube.

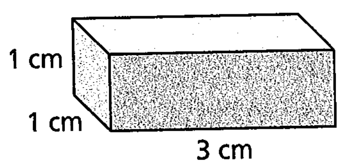
A. How many different nets can you make that will fold into a box shaped like a unit cube?

B. What is the total area of each net in square units?

ACE Homework starts on page 10.

1.2 Making Rectangular Boxes

Many boxes are not shaped like cubes. The rectangular box below has square ends, but the remaining faces are non-square rectangles.



Problem 1.2 Making Rectangular Boxes

A. On grid paper, draw two different nets for the rectangular box above. Cut each pattern out and fold it into a box.

B. Describe the faces of the box formed from each net you made. What are the dimensions of each face?

C. Find the total area of each net you made in Question A.

D. How many centimeter cubes will fit into the box formed from each net you made? Explain your reasoning.

E. Suppose you stand the rectangular 1 centimeter × 1 centimeter × 3 centimeters box on its end. Does the area of a net for the box or the number of cubes needed to fill the box change?

active math **online**
For: Virtual Box Activity
Visit: PHSchool.com
Web Code: and-6102

ACE Homework starts on page 10.

1.3 Testing Nets

At a Glance

PACING 1 day

Mathematical Goals (Objectives)

- Visualize a net as a representation of the surface area of a rectangular prism
- Use a net for a rectangular prism to develop a strategy for finding the surface area of the prism
- Find the volume of a rectangular prism by counting the number of unit cubes it takes to fill the prism

Launch

Like working backwards.

Tell the story of the engineer who has lost his notes indicating the dimensions of each box. Distribute Labsheet 1.3 to each student. Before students begin cutting out the nets, ask them to guess and record the dimensions of each box. This will help strengthen their visualization skills. Students can work in groups of 2–3.

"What are the prisms dimensions?"

Use an example of a 2 by 5 by 8 prism
"What are the dimensions of each face"
"What patterns do you notice?

Materials

- Transparencies 1.3A and 1.3B
- Labsheet 1.3
- Scissors
- cm cubes

Vocabulary

- rectangular prism
- base

Explore

Before students cut they should draw in the fold lines

Have students cut out the nets and then find the dimensions and the number of unit cubes needed to fill each box. Have students fold the nets so the squares are on the outside of the box. (Easier to count to find area)

You might want to ask some students to cut out their nets in Question E to share during the summary.

If students are having trouble finding area when its folded, unfold it and let them count.

Summarize

SHARE

Ask students to explain how they decided where to fold each net. Some will have used the symmetry of the two pieces that "stick out" as the place to begin folding.

Discuss the dimensions of each box. Emphasize that the faces of a box come in pairs—this will be an important idea when students develop strategies for finding surface area. Stress the importance of the base, its dimensions, and the height (the distance from the base to the top of the box).

Steer kids away from finding rules at this point (Discrete!)

When discussing the number of unit cubes needed to fill each box, do not go for rules. The rule or formula for finding surface area and volume will be developed in the next investigation.

Have students share their solutions to Question C. As each student displays his or her net and tells the class its dimensions and its area, ask the class whether they agree that the net works. Also ask how the net, its dimensions, and its area compare to those for Box P.

SHARE

Materials

- Student notebooks

ACE Assignment Guide for Problem 1.3

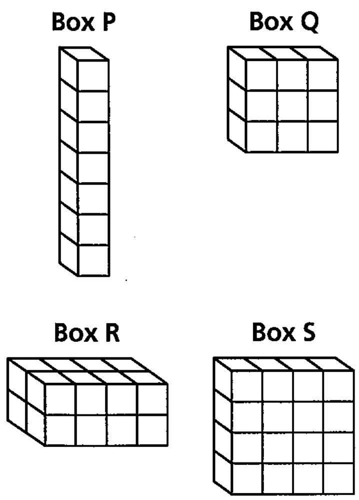
Differentiated Instruction
Solutions for All Learners

Core 8–9
Other *Connections* 25–27, 31

Adapted For suggestions about adapting Exercise 7 and other ACE exercises, see the CMP *Special Needs Handbook.*
Connecting to Prior Units 25, 26: *Covering and Surrounding*; 27: *Bits and Pieces II*; 31: *Bits and Pieces III*

Answers to Problem 1.3

A.



Box P: 1 cm by 1 cm by 6 cm;
Box Q: 1 cm by 3 cm by 3 cm;
Box R: 2 cm by 2 cm by 4 cm;
Box S: 1 cm by 4 cm by 4 cm.

B. Each combination of two dimensions will yield the dimensions for a pair of congruent faces.

C. Box P: 26 cm^2; Box Q: 30 cm^2; Box R: 40 cm^2; Box S: 48 cm^2.

D. Box P: 6 unit cubes; Box Q: 9 unit cubes; Box R: 16 unit cubes; Box S: 16 unit cubes.

E. Answers will vary. The box should hold 6 unit cubes and have dimensions 1 cm by 2 cm by 3 cm.

Questions to ask
"What features of the box make it easier for you to find the surface area?"

Clarify: height = distance from the base to the top of the box.

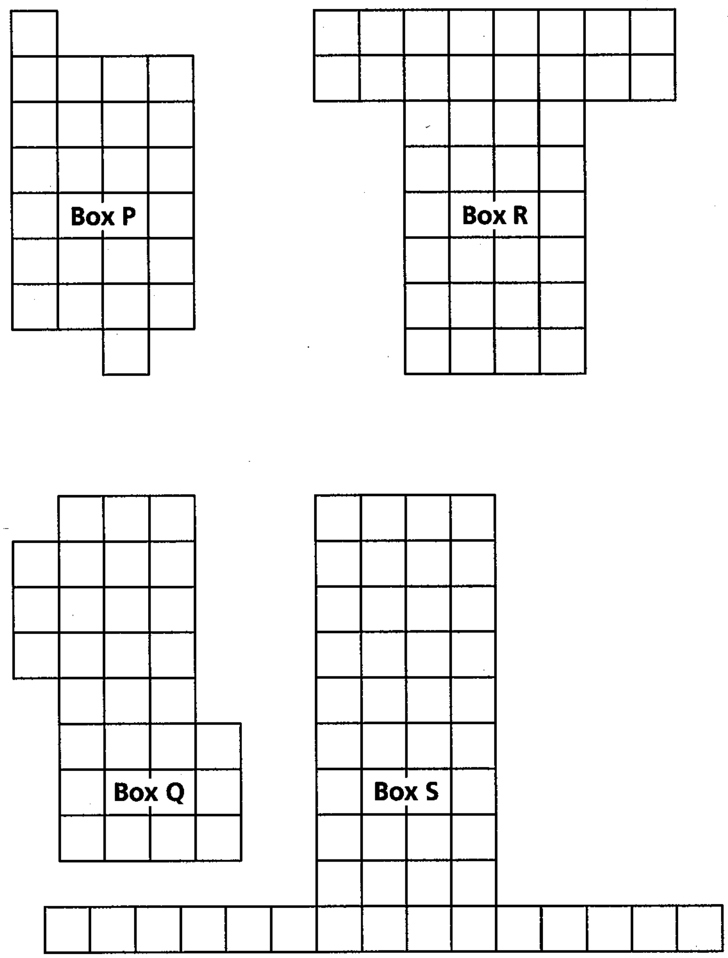
Some students might count individual boxes needed to fill, others may count in layers.

Name ____________________ Date __________ Class __________

Labsheet 1.3

Filling and Wrapping

Box Nets



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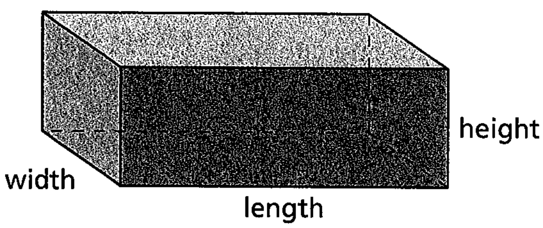
1.3 Testing Nets

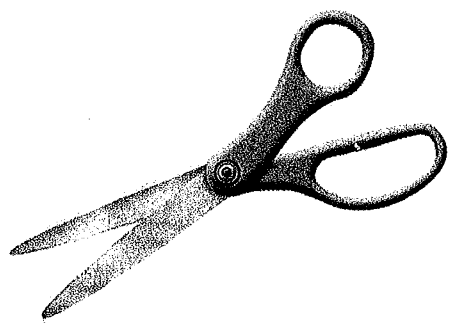
All the boxes you have made so far are rectangular prisms. A **rectangular prism** is a three-dimensional shape with six rectangular faces. The size of a rectangular prism can be described by giving its *dimensions*. The dimensions are the length, the width, and the height.

The **base** of a rectangular prism is the face on the bottom (the face that rests on the table or floor). The length and width of a prism are the length and width of its rectangular base. The height is the distance from the base of the prism to its top.

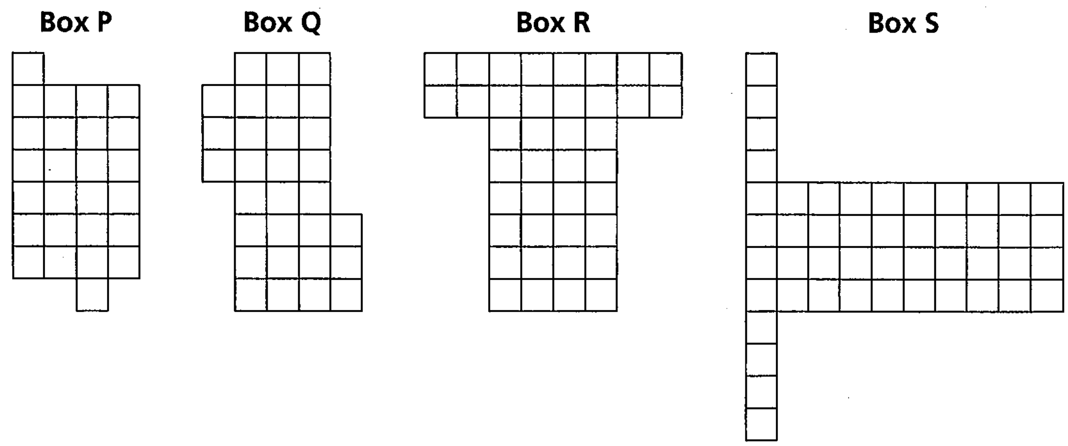
Getting Ready for Problem 1.3

- Suppose you want to cut the box in the figure below to make a net for the box. Along which edges can you make the cut?
- Are there different choices of edges to cut that will work?





An engineer at the Save-a-Tree packaging company drew the nets below. He lost the notes that indicated the dimensions of the boxes. Use your thinking from the Getting Ready section to work backwards and determine the dimensions for him.



Problem 1.3 Rectangular Prisms

A. Using a copy of the diagram above, draw in fold lines and cut each pattern and fold it to form a box. What are the dimensions of each box?

B. How are the dimensions of each box related to the dimensions of its faces?

C. What is the total area, in square units, of all the faces of each box?

D. Fill each box with unit cubes. How many unit cubes does it take to fill each box?

E. Design a net for a box that has a different shape than Box P but holds the same number of cubes as Box P.

ACE **Homework starts on page 10.**

1.4 Flattening a Box

Amy is a packaging engineer at the Save-a-Tree packaging company. Mr. Shu asks Amy to come to his class and explain her job to his students. She gives each student a box to do some exploring.

At a Glance

2.1 Packaging Blocks

PACING $1\frac{1}{2}$ days

Mathematical Goals (Objectives)

- Connect the dimensions of a rectangular prism to its volume and surface area
- Understand that rectangular prisms may have the same volume but quite different surface areas

Launch

use cubes

Use the nets and boxes from Investigation 1 to demonstrate the concepts of volume and surface area.

- *How many unit cubes fit inside of box R?*
- *The word for the number of unit cubes that fill a solid is* volume. *What is the volume of box S?*
- *So boxes R and S have the same volume. What is different about these two boxes?*
- *We call the sum of the areas of the faces the* surface area *of the solid. Which box has a larger surface area, R or S?*

Tell the story of ATC Toy Company. Before students break into groups, have the class suggest one arrangement of 24 blocks and discuss how they might find its surface area. If you want students to organize their data in a table (as shown in the student edition), model the process by entering the data about the chosen arrangement into a table. Or, let students decide how to organize their work to look for patterns.

before modeling let students Think-Pair-Share to come up with 1 row entry so teacher isn't giving away answers.

You may suggest that students make sketches for only ~~one or two of the boxes~~ some boxes. Choose 4 to make sketches, etc. . .

Have students work in groups of two to four. table groups

Materials

- Inch cubes or other unit cubes
- Transparency 2.1
- Nets and boxes from Investigation 1

Vocabulary

- volume
- surface area

Explore

Encourage students to organize their information in a table as suggested in the problem or in some other way that makes sense to them. They should sketch each arrangement they find and label its dimensions.

As you listen to students talk and ask them questions, encourage the use of the vocabulary: surface area and volume. *IMPORTANT*

Summarize

SHARE

have table groups share data.

Begin the summary by collecting students' data.

- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 1?* What is the length, width, height?
- *How much material will it take to cover this box?*
- *Did anyone find a box that holds exactly 24 cubes and has an edge length of 2?* What is the length, width, height?

Continue with this line of questioning. Discuss identical arrangements.

Materials

- Student notebooks

continued on next page

Review - Reinforce volume = units that fill a solid
surface area = add area of faces together.

Summarize

continued

- *How did you decide which face to use for the base? Does your choice affect the surface area of the box?* (No!)

"As a dimension increases what do you notice?"

- Have students describe the patterns they see in the table.

Discuss which box has the least surface area, which has the greatest, and what these boxes look like.

ACE Assignment Guide for Problem 2.1

Differentiated Instruction
Solutions for All Learners

Core 1–3, 20

Other *Connections* 21, 22

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 20: *Prime Time*; 22: *Variables and Patterns*

Answers to Problem 2.1

A. (Note: Students' sketches may show the same arrangement in a different orientation.)

(Figure 1)

B. The 4-by-3-by-2 box requires the least amount of material. The 24-by-1-by-1 box requires the most material.

C. Possible answer: ATC Toy Company should use the 4-by-3-by-2 box because it has the least surface area (52 in.^{2}) and would therefore be the least expensive to buy or to make. (Note: The box shaped most like a cube will always have the least surface area. This is pursued in more depth in Problem 2.2. Don't expect your class to make this generalization at this time.)

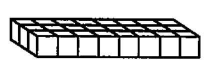
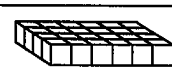
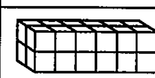
Some students may argue for boxes based on their visual appeal to the buyer.

D. Possible answer: Because 24 has more factors than 26, there are more ways to effectively package 24 blocks. With 26 blocks, you only have a 1-by-1-by-26 or a 1-by-2-by-13 arrangement. These two boxes are long and thin, so they will have larger surface areas than if the box could be more cubic in shape, like you can get with 24 blocks.

Some students might argue that the lesser-used letters don't need their own blocks, so the company can economize by making sets of 24 instead of 26.

Figure 1

Possible Arrangements of 24 Cubes

Length (in.)	Width (in.)	Height (in.)	Vol (in.^{3})	Surface areas (in.^{2})	Sketch
24	1	1	24	98	
12	2	1	24	76	
8	3	1	24	70	
6	4	1	24	68	
6	2	2	24	56	
4	3	2	24	52	

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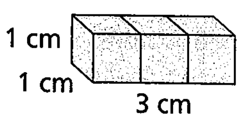
Investigation 2

Designing Rectangular Boxes

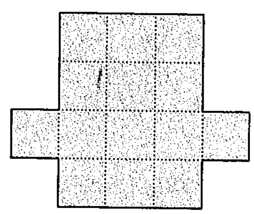
Finding the right box for a product requires thought and planning. A company must consider how much the box can hold as well as the amount and the cost of the material needed to make the box.

The amount that a box can hold depends on its volume. The **volume** of a box is the number of unit cubes that it would take to fill the box. The amount of material needed to make or to cover a box depends on its surface area. The **surface area** of a box is the total area of all of its faces.

The box shown below has dimensions of 1 centimeter by 3 centimeters by 1 centimeter. It would take three 1-centimeter cubes to fill this box, so the box has a volume of 3 cubic centimeters. Because the net for the box takes fourteen 1-centimeter grid squares to make the box, the box has a surface area of 14 square centimeters.



volume = 3 cubic centimeters



surface area = 14 square centimeters

In this investigation, you will explore the possible surface areas for a rectangular box that holds a given volume.

2.1 Packaging Blocks

ATC Toy Company is planning to market a set of children's alphabet blocks. Each block is a cube with 1-inch edges, so each block has a volume of 1 cubic inch.

Problem 2.1 Finding Surface Area

The company wants to arrange 24 blocks in the shape of a rectangular prism and then package them in a box that exactly fits the prism.

A. Find all the ways 24 cubes can be arranged into a rectangular prism. Make a sketch of each arrangement. Record the dimensions and surface area. It may help to organize your findings into a table like the one below:

Possible Arrangements of 24 Cubes

Length	Width	Height	Volume	Surface Area	Sketch

B. Which of your arrangements requires the box made with the least material? Which requires the box made with the most material?

C. Which arrangement would you recommend to ATC Toy Company? Explain why.

D. Why do you think the company makes 24 alphabet blocks rather than 26?

ACE **Homework starts on page 24.**

2.2 Saving Trees

At a Glance

PACING 1 day

Mathematical Goals (Objectives)

- Predict which rectangular prism of those with a common volume will have the smallest surface area
- Refine a strategy for finding the surface area of a rectangular prism

Launch

Review what students discovered in Problem 2.1.

- *How would you describe the shape of the box we found in the last problem that held 24 cubes and had the least amount of surface area?*

Introduce Problem 2.2.

In mathematics, we are always looking for patterns and rules that will help us to predict outcomes. In today's problem, you are challenged to explore prisms with different volumes. You are asked to look carefully at the data and make conjectures about what you think will help you to predict the arrangement with the smallest surface area.

Let the class work on the problem in groups of 3 or 4.

"How does this difference show up in the dimensions of the boxes?"

Materials

- Inch cubes
- 2 or 3 rectangular or cubic boxes

Explore

Encourage groups who make conjectures about the arrangement of cubes that requires the least amount of packaging material to test other arrangements of the same number of cubes.

Test your conjectures on a number of cubes other than the 8, 27, and 12 suggested in the problem.

group will have to organize data, check in to verify!

If a group is having trouble with the problem, talk through the case of eight cubes with them. Ask them to build each arrangement and to look at the physical objects as well as the measures in their table.

Look at the dimensions for each arrangement and how they change from one arrangement to another.

- *What is the difference between the box with the greatest surface area and the box with the least surface area?*
- *How does this difference show up in the actual boxes made from cubes?*
- *How does this difference show up in the dimensions of the boxes?*

Summarize

SHARE

Ask students to explain why the more cube-like rectangular arrangement requires the least packaging material.

Put the boxes with the greatest surface area together and those with the least surface area together.

- *How would you describe these shapes compared to these?*

Materials

- Student notebooks

continued on next page

Summarize *continued*

"How would you compare these shapes to these?"

- *Why is the more cube-like rectangular box the box with the least surface area?*

Have students describe their processes for finding surface area. Model a symbolic representation of their strategies. This is not to develop a formula that students need to memorize. Instead, it is to encourage the kind of careful thinking required to write a formula.

Help the class further explore the minimal surface area.

ACE Assignment Guide for Problem 2.2

Differentiated Instruction
Solutions for All Learners

Core 4–6

Other *Connections* 23–24; *Extension* 28; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 5 and other ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 23–24: *Covering and Surrounding*

Answers to Problem 2.2

A. The rectangular arrangements of cubes with the least surface area are:

1. 8 cubes: 2 by 2 by 2 (surface area: 24 in.^{3})
2. 27 cubes: 3 by 3 by 3 (surface area: 54 in.^{3})
3. 12 cubes: 2 by 2 by 3 (surface area: 32 in.^{3})

B. **1.** Possible true conjecture: The rectangular arrangement of a given number of cubes with the least surface area is the one that is most like a cube. Students may also use language like *compact* or *shortest* (in contrast to the long, skinny packages). This is similar to the conjecture that students examined in *Covering and Surrounding*: that the rectangle with the smallest perimeter for a given area is a square.

2. The conjecture in B gives:

30 cubes: 2 by 3 by 5

64 cubes: 4 by 4 by 4, each of which has the smallest surface area for the given number of cubes.

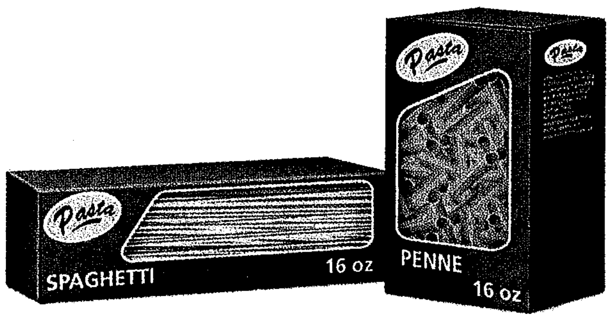
One way to think about justifying the conjecture is that the exposed faces of the small cubes generate the surface area. The more compact (or cube-like) the prism, the more faces of the small cubes face the interior of the prism, so fewer are exposed as surface area.

C. Possible answers: Find the area of each of the six faces and add them together. Find the area of the front, the top, and the right side; add these together and double the answer.

2.2 Saving Trees

You discovered that 24 blocks can be packaged in different ways that use varying amounts of packaging material. By using less material, a company can save money, reduce waste, and conserve natural resources.

Which rectangular arrangement of cubes uses the least amount of packaging material?



Problem 2.2 Finding the Least Surface Area

A. Explore the possible arrangements of each of the following numbers of cubes. Find the arrangement that requires the least amount of packaging material.

1. 8 cubes **2.** 27 cubes **3.** 12 cubes

B. 1. Make a conjecture about the rectangular arrangement of cubes that requires the least packaging material.

2. Does your conjecture work for 30 cubes? Does it work for 64 cubes? If not, change your conjecture so it works for any number of cubes. When you have a conjecture that you think is correct, give reasons why you think your conjecture is valid.

C. Describe a strategy for finding the total surface area of a closed box.

ACE **Homework starts on page 24.**

2.3 Filling Rectangular Boxes

PACING 1 day

Mathematical Goals (Objectives)

- Understand that prisms can be filled systematically in identical layers, and that this layering leads to the formula for volume
- Develop a formula for finding the volume of a rectangular prism

Launch

Construct boxes from grids.

Discuss the Getting Ready.

Talk about Save-a-Tree's ready-made box sizes and ATC Toy Company's decision. Hold up a box.

- *What is the volume of my box? How did you make your estimate?*

Ask students to estimate the volume of each box. Record some of the estimates on the board. Tell the class that the intent of this problem is for them to look for efficient ways to find the volume of a box. If some students claim that they already have a rule for finding the volume of a box (volume = $\ell \times w \times h$), question them about it.

- *What does your rule mean?*
- *Why do you think it will work?*
- *Will it work for all prisms?*

Students can work in pairs.

Materials
- Pre-made box for demonstration
- Transparent grids (optional)
- Transparent models of Boxes W, X, Y, and Z (optional)
- Transparencies 2.3A and 2.3B

Explore

Remind students to save the transparent boxes for the summary. Some students may need cubes to simulate filling the boxes.

You may want to suggest that students organize their work in a table.

As students make progress in their pairs, ask them how close their estimates of the volume were to the answers they are finding.

Materials
- Inch cubes
- Inch or other grid paper

Summarize

SHARE

Discuss the answers to Question A. If some students offer the formula volume = $\ell \times w \times h$, ask what this means in terms of counting layers. Talk about the answers to Question B. Spend time on students' strategies for finding the surface area of a box.

Ask some questions to probe students' understanding.

- *Why is the number of cubes in the bottom layer equal to the area of the base?*

If you have constructed models of the boxes, hold one of them up at the orientation shown in the problem.

- *What are the dimensions of the base of this box? What is its height? What is the volume of the box?*

Materials
- Student notebooks
- Transparency 2.3C

continued on next page

Guide students to visualize the surface area as the area of the six faces. (Pairs of faces in a rectangular prism will streamline process)

Summarize *continued*

Now, set the box on a different base.

- *What is the area of the new base? How many cubes will fit on the base? How many layers will be needed to fill the box?*
- *Does this new orientation change the volume? What is the surface area of this box? Would changing the orientation of the box change its surface area?*

Go over the volumes and surface areas of the other boxes.

→ Students share, teacher corrects misconceptions.

ACE Assignment Guide for Problem 2.3

Differentiated Instruction
Solutions for All Learners

Core 8–15

Other *Applications* 7, 16–19; *Connections* 25–27; *Extensions* 29; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 2.3

A. **1.** 8 cubes

2. 10 layers

3. 80 cubes

4. See the Summarize section for some possible explanations.

B. 136 $in.^2$

C. The volume doesn't change. The number of cubes in the first layer changes, but so does the number of layers. The volume is: area of base × height, or 10 × 4 × 2. In the original position, the volume was 2 × 4 × 10. Similarly, the surface area of the box does not change.

D. Surface area: Box W: 52 $in.^2$; Box X: 54 $in.^2$; Box Y: 132 $in.^2$

Volume: Box W: 24 cubes; Box X: 27 cubes; Box Y: 80 cubes

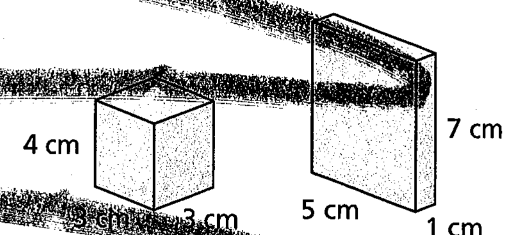
Did You Know?

Area is expressed in square units, such as square inches or square centimeters. You can abbreviate square units by writing the abbreviation for the unit followed by a raised 2. For example, an abbreviation for square inches is in^2.

Volume is expressed in cubic units. You can abbreviate cubic units by writing the abbreviation for the unit followed by a raised 3. For example, an abbreviation for cubic centimeters is cm^3.

Getting Ready for Problem 2.3

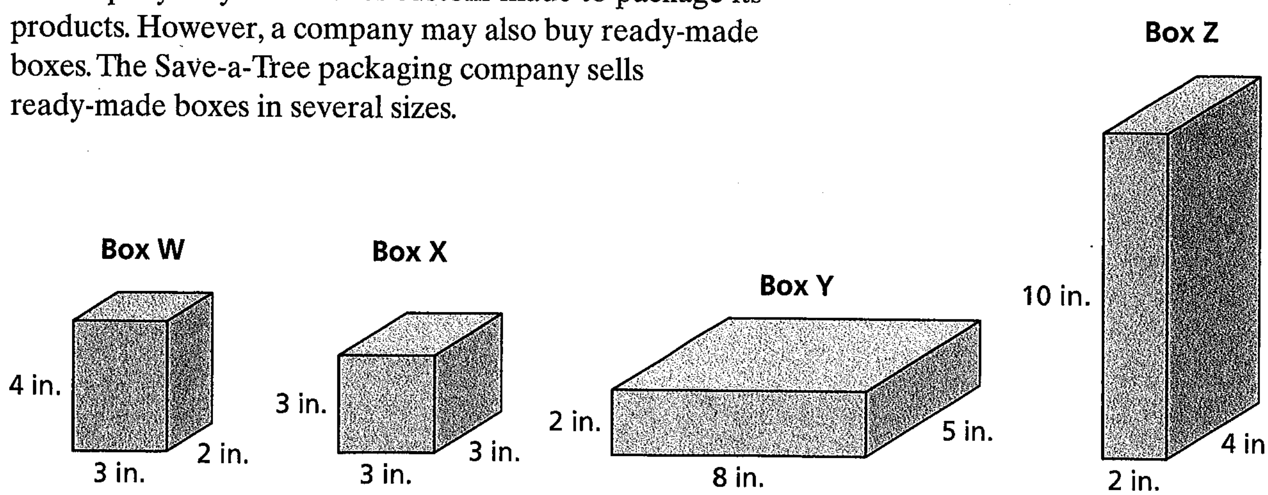
One seventh-grade student, Bernie, wonders if he can compare volumes without having to calculate them exactly. He figures that volume measures the contents of a container. He fills the prism on the left with rice and then pours the rice into the one on the right.



- How can you decide if there is enough rice or too much rice to fill the prism on the right?

2.3 Filling Rectangular Boxes

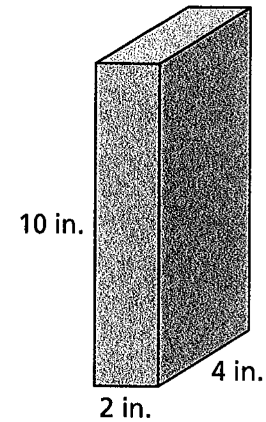
A company may have boxes custom-made to package its products. However, a company may also buy ready-made boxes. The Save-a-Tree packaging company sells ready-made boxes in several sizes.



Problem 2.3 Finding the Volume of a Rectangular Prism

ATC Toy Company is considering using Save-a-Tree's Box Z to ship alphabet blocks. Each block is a 1-inch cube. ATC needs to know how many blocks will fit into Box Z and the surface area of the box.

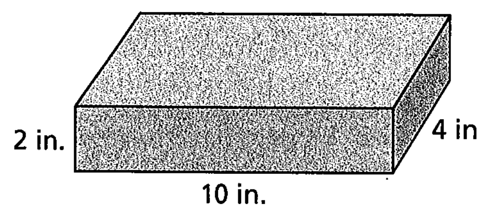
A. The number of unit cubes that fit in a box is the volume of the box.



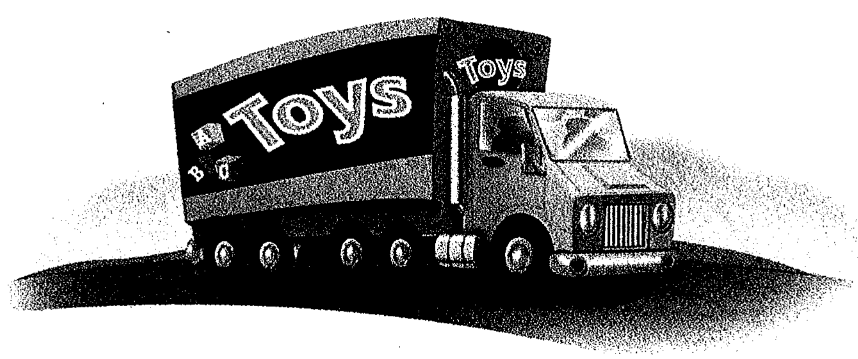
1. How many cubes will fit in a single layer at the bottom of this box?
2. How many identical layers can be stacked in this box?
3. What is the total number of cubes that can be packed in this box?
4. Consider the number of cubes in each layer, the number of layers, the volume, and the dimensions of the box. What connections do you see among these measurements?

B. Find the surface area of Box Z.

C. Suppose Box Z is put down on its side so its base is 4 inches by 10 inches and its height is 2 inches. Does this affect the volume of the box? Does this affect the surface area? Explain your reasoning.



D. Apply your strategies for finding volume and surface area to Boxes W, X, and Y.



ACE Homework starts on page 24.

Lesson 14: The Locker Game

Mathematical Goals (Objective):

- Determine attributes of numbers and their factors

Launch:

Introduce students to the locker game.

The 20 students in Steve's 6th grade class are playing a game in a hallway that is lined with 20 lockers in a row.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

- The first student starts with the first locker and goes down the hallway and opens all the lockers.
- The second student starts with the second locker and goes down the hallway and shuts every other locker.
- The third student stops at every third locker and opens the locker if it is closed or closes the locker if it is open.
- The fourth student stops at every fourth locker and opens the locker if it is closed or closes the locker if it is open.

This process continues until all 20 students in the class have passed through the hallway.

Who Touched What?	
Student Number	Locker #'s They Touched
1	
2	
3	
4	
5	
6	

7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Explore:

When students have completed the initial discussion for the "Who touched what?" question, encourage them to continue to investigate the following questions in groups of 4:

a. Which lockers are still open at the end of the game? Explain your reasoning.
b. Which lockers were touched the most?
c. Which lockers were touched by only three students? Explain your reasoning.
d. Which lockers were touched by only two students? Explain your reasoning.

Share:

Students will share the procedure they followed to reach their solution. The Teacher will guide groups to share what patterns emerged in their work.

- Lockers touched the most have the most factors.
- Lockers touched by 3 students are squares of primes and have 3 factors.
- Lockers touched by 2 students are prime and have 2 factors.

- How can we tell if a # is prime?
- What are examples of prime numbers?

Summarize:

The teacher will review with students that we can tell a number is prime if it has only two factors (1 and itself). Ask each group for an example of a prime number and the strategies they have used to find the prime numbers. If needed guide students towards tree diagrams.

Teacher will summarize:

- Lockers touched the most have the most factors.
- Lockers touched by 3 students are squares of primes and have 3 factors.
- Lockers touched by 2 students are prime and have 2 factors.
  - The first locker will...
  - Prime numbered lockers will...
  - Composite numbered lockers will...

Lesson 15: Hotdog-Hotdog-Hotdiggity-dog

Mathematical Goals (Objective):

- Students will develop strategies to determine the least common multiple of two numbers

Launch:

Introduce the students to George Banks. He is a little stressed out. His daughter is getting married. He has a lot of people coming to town for the wedding and some are staying at his house. He is running late and needs to finish grocery shopping so he can cook everyone dinner.

Play the students the hotdog bun clip from the movie "Father of the Bride."

Ask students what might happen if you buy only one package of hot dogs and one package of buns? How might you feel? Tell them that their challenge is to decide how many people George can feed at his house if everyone gets one hot dog with no extra hotdogs or hotdog buns leftover. How many packages of hotdog buns should he purchase? Hotdogs? How many people can eat at his house?

Explore:

Students will work in groups of 4 to solve the problem. The teacher may need to remind students about the unit they are working in (single hotdog bun combos vs package combos). Listen for strategies students develop.

Share:

Student groups will share the strategies they followed to reach their solution. Some strategies should include listing factors and completing factor trees. If needed the teacher should guide students toward the factor tree as a way to locate least common multiples.

The class will discuss which strategies are the most efficient and situations where you might choose one strategy over another to use.

Summarize:

The teacher will review the different strategies shared by the class. The least common multiple is the smallest number divisible by the pair of numbers. LCMs will be helpful when reducing fractions.

Name: ________________________

Discrete Math Unit Post-Test

1. What is the difference between area and perimeter?

Use figure A to answer the questions 2 and 3.

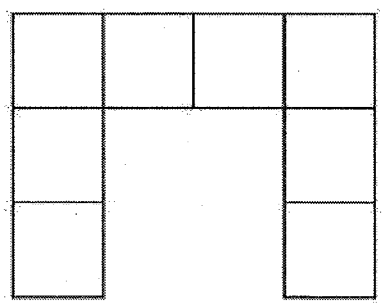


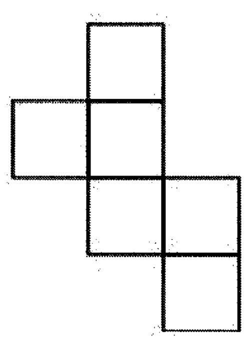
Figure A

2. What is the area of figure B? Label your answer. ________________________

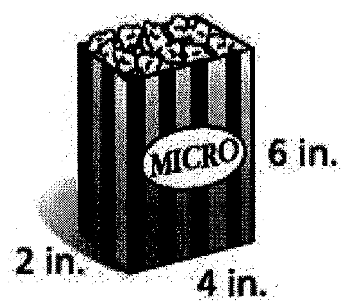
3. What is the perimeter of Figure B? Label your answer. ____________________

4. Show how you can move one time to change the perimeter to 16. Mark figure A to show which tile you moved and to where.

5. Could the net below be folded along the lines to form a box? If yes, explain how. If no, explain why not.



6. The Apple Theater concession sells two sizes of popcorn--a micro box and a jumbo box. About how many square inches of cardboard are needed to make the microbox? (there is no top on the box). You may use cubes if you would like.



7. A game is played by rolling a four-sided number cube with faces numbered 1, 2, 3, 4 and a six-sided number cube with faces numbered 1, 2, 3, 4, 5, 6 and finding the sum of the numbers rolled.

A player wins by rolling a sum of 2, 3, 4, 9, or 10; otherwise, the player loses.

a. List all the possible number pairs that can be rolled and find the sum of each pair.

B. Is this game fair or unfair? Use your outcome list to explain

Final Project Format

Group members: Lisa Fisher & Sarah Winger

Grade level: 6th   Unit title: Discrete math

Page 1 – **Title page**: Should contain the unit title, group members (including school, and email address), and appropriate grade level(s).

Page 2 – **Executive summary**: This page should provide an overview of the 15 day unit you are preparing. The overview should include the specific Minnesota State Math Standard(s) you are addressing. You should also briefly explain the various learning opportunities you are going to lead the students through in order to reach the standard(s) you have previously identified as the goal of this unit. You should identify one or more sample MCA (Minnesota Comprehensive Assessment) question(s) that your students will be able to successfully answer because of this unit being taught to them.

Page 3 – **Table of Contents**

Page 4 – **Lessons**: Each lesson should begin with the objective (related to MN Standards) of the lesson. The following format should be used as you work through the lesson.

❖ **Launch**: This is where the teacher sets the context of the problem or activity being worked on this day. This involves making sure the students clearly understand the mathematical context and the mathematical challenge of the day's activities.

❖ **Explore:** This is the time where the students get to work in pairs, individually, or as a class to solve problems presented by the lesson.

❖ **Share:** This occurs when most of the students have made sufficient progress toward solving the problem presented with today's lesson. It is during this phase that the students learn how others approached the problem and possible solution routes. Helps students deepen their understanding of the mathematical ideas presented in the day's lesson.

❖ **Summarize**: During this phase the teacher concludes the lesson by clearly stating what the main idea was in the lesson, being sure to clear up any confusion that may arise during the "share" segment. Helps students focus their understanding of the mathematical ideas presented in the lesson.

Remember to include appropriate citations and scans of game boards and other manipulatives. If in doubt then include a citation. Also remember the unit does not have to be tied to a specific text.

A first draft of the project needs to be perused by Craig or Todd.

First draft checked by: CR
(initialed)

A final draft of the project needs to be initialed by the **other** person than checked the first draft.

Final draft checked by: Todd J.
(initialed)